22P404 (Pages: 2) Name: ……………………….…..

Reg. No……………………..

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH4 E09 – DIFFERENTIAL GEOMETRY

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage

1. Show that graph of any function f: $R^n \to R$ is a level set for some function F : $R^{n+1} \to R$.
2. Find the divergence of the vector field **X**(P) = (P, -P) on R^2.
3. State Lagrange's Multiplier Theorem.
4. Prove that geodesics have constant speed.
5. Define covariant derivative of a smooth vector field along a parametrized curve.
6. Define Weingarten map L_p: $S_p \to S_q$
7. Let S be an n- surface in R^{n+1} and let p ∈ S. Define the first and second fundamental forms of S at p.
8. State Inverse function theorem for a smooth map $\varphi: S \to \bar{S}$ where S and $\bar{S}$ are n-surfaces.

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. Find an integral curve through p(1,1) of the vector field X on R^2 given by $\boldsymbol{X}(x_1,x_2)$ = $(x_2, -x_1)$
10. Let S be the unit circle $x_1^2+ x_2^2 = 1$. Define g : $R^2 \to R$ by $g(x_1,x_2) = a\, x_1^2+ 2bx_1\, x_2+c\, x_2^2$ where a, b, c ϵ R. Find the extreme points and extreme values of g on S.
11. Let S ⊆ R^{n+1} be a connected n-surface. Then prove that there exist on S exactly two smooth unit normal vector fields.

UNIT II

12. Let S denote the unit sphere $x_1^2+ x_2^2 +x_3^2 = 1$ in R^3. Show that $\alpha(t) =$ (cosat, sinat, 0) for some a ∈ R and a ≠ 0 is a geodesic of S.

13. Prove that Wiengarten map L_p is self adjoint.

14. Find the global parametrization of the curve $(x_1-a)^2+(x_2-b)^2 = r^2$ oriented by $\frac{\nabla f}{\|\nabla f\|}$.

UNIT III

15. Describe the relation between Gauss- Kronecker curvature and the principal curvatures of an n- surface

16. Show that on each compact oriented n- surface S there is a point $p \in S$ such that second fundamental form at p is definite.

17. Let S be an n-surface in R^{n+1} and let $f : S \rightarrow R^k$. Prove that f is smooth if and only if $f o \varphi$ is smooth for each local parametrization $\varphi : U \rightarrow S$.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let S be a compact, connected oriented n- surface in R^{n+1}. Prove that the Gauss map maps S onto the unit sphere S^n.

19. Let S be an n-surface in R^{n+1}, let $p \in S$ and $v \in S_p$. Then prove the existence and uniqueness of the maximal geodesic in S passing through p with initial velocity v.

20. Let C be an oriented plane curve. Prove that there exist a global parametrization of C if and only if C is connected.

21. Find the Gaussian curvature of the ellipsoid $\frac{x_1^2}{a^2}+\frac{x_2^2}{b^2}+\frac{x_3^2}{c^2}=1$

(2 × 5 = 10 Weightage)

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