23U210S   (Pages: 2)   Name ……………………….

                                                          Reg. No: ..…………………..

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024
(CUCBCSS-UG)
CC15U MAT2 C02 – MATHEMATICS
(Mathematics - Complementary Course)
(2015 to 2018 Admissions - Supplementary)

Time: Three Hours   Maximum: 80 Marks

PART A
Answer *all* questions. Each question has 1 mark.

1. Define a partition of $[a, b]$.

2. Find the norm of the partition $P = \{0, 1.2, 1.5, \ 2.3, 2.6, 3\}$

3. $\frac{d}{dx}\left(\int_0^x \frac{1}{1+t^2} dt\right)$ = ……………..

4. Set up an integral for the area of the surface generated by revolving the curve

   $y = \tan x$, $0 \leq x \leq \frac{\pi}{4}$ about the x- axis.

5. $\cosh 2x = $ ………….

6. Show that $\operatorname{csch}^{-1} x = \sinh^{-1}\frac{1}{x}$

7. Find a formula for the nth term of the sequence $0,1,1,2,2,3,3,4,...$

8. Find the sum of the series $\sum_{n=1}^{\infty}(-1)^n \frac{7}{4^n}$.

9. State the absolute convergence test.

10. Graph the set of points whose polar coordinates satisfy the conditions $\frac{2\pi}{3} \leq \theta \leq \frac{5\pi}{6}$

11. Identify the conic $r = \frac{6}{2+3\cos\theta}$.

12. Find the slope of the cardiod $r = 1 + \cos\theta$ at $\theta = 0$.

                                                          (12 × 1 = 12 Marks)

PART B
Answer any *seven* questions. Each question has 2 marks.

13. Without evaluating show that the value of $\int_0^1 \sqrt{1 + \cos x}\, dx$ is less than $3/2$

14. State mean value theorem for definite integrals.

15. Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^2\theta \cot\theta\, d\theta$.

16. Find the volume of the torus (doughnut) generated by revolving a circular disk of radius
    a about an axis in its plane at a distance $b \geq a$ from its center.

17. Show that $\cosh^2 x - \sinh^2 x = 1$.

18. Show that the alternating harmonic series converges conditionally.

19. Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $x = 2$.

20. Find the equivalent Cartesian point corresponding to a point whose polar coordinate is given by $\left(5, \tan^{-1}\left(\frac{4}{3}\right)\right)$.

21. Find the area of the region in the plane enclosed by the cardiod $r = a(1 + \cos\theta)$.

(7 × 2 = 14 Marks)

PART C
Answer any *six* questions. Each question has 5 marks.

22. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x$, $-1 \le x \le 2$.

23. A pyramid 3 m high has a square base that is 3 m on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

24. Find the length of the curve $x = \frac{y^4}{4} + \frac{1}{8y^2}$ from $y = 1$ to $y = 2$.

25. Evaluate $\int_{-\ln 2}^{0} \cosh^2\left(\frac{x}{2}\right) dx$.

26. Discuss the convergence of $\{a_n\}$ with $a_n = \frac{\ln n}{n^{1/n}}$.

27. Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$.

28. Find the points of intersection of the curves $r^2 = 4\cos\theta$ and $r = 1 - \cos\theta$.

29. Find the area inside the smaller loop of the limacon $r = 2\cos\theta + 1$.

(6 × 5 = 30 Marks)

PART D
Answer any *three* questions. Each question has 8 marks.

30. (i) Find the mean value of $f(x) = \sqrt{4 - x^2}$ on $[-2,2]$.

   (ii) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $x + y = 0$.

31. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, \ 1 \le y \le 2$, about the x-axis.

32. Discuss the convergence of the alternating series $\sum \frac{(-1)^n}{1+\sqrt{n}}$.

33. Find the radius of convergence and interval of convergence of the power series
   $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$

34. Find the area of the surface generated by revolving the right hand loop of $r^2 = \cos 2\theta$ about the y-axis.

(3 × 8 = 24 Marks)

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