23U201 (Pages: 2) Name: .....

Reg.No: .....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS2 B02 / CC20U MTS2 B02 - CALCULUS OF SINGLE VARIABLE - I

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Evaluate $lim_{x\to 4} t^{\frac{-1}{2}}(t^2-3t+4)^{\frac{3}{2}}$.
2. Define limit of a function at a number(Precise definition).
3. State Intermediate Value Theorem.
4. Define Tangent line at a point P on the graph of a function f.
5. Find the derivative of the function $f(x)=x+\sqrt{x}$.What is its domain?
6. Define accelaration and jerk of a body having position function $f(x)$.
7. Assume that the moon is a perfect sphere and suppose that we have measured its radius and found it to be $1080mi$ with a possible error of $0.05mi$. Estimate the maximum error in the computed surface area of the moon
8. Find the linearization of $f(x)=x^3+2x^2$ at $a=1$.
9. Define absolute maximum at a point and the maximum value. Explain it with a map.
10. Using Mean value theorem verify the function f(x) = sin x ; [0, π/2] and find c.
11. The vertical asymptote of the graph of$f(x)=1/(x-1)$
12. Find two positive numbers whose sum is 100 and whose product is a maximum.
13. Evaluate $\int \frac{x^2-2x+3}{\sqrt{x}}dx$.
14. Find the work done in lifting a $50-lb$ sack of potatoes to a height of $4ft$ above the ground
15. Find the center of mass of a system of four objects located at the points -3, -1, 2 and 4, on the x-axis (x in meters), with masses 3, 2, 4 and 6 kilograms respectively

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Let $f(x) = \frac{1}{x+1}$ and$g(x) = \frac{x}{x+1}$.Find $f+g, f-g, fg, f/g.$

17. Find $\lim_{x\to 0} \sin(\frac{1}{x})$.

18. (a) Find the points of inflection of f(x) = x^4 - $4x^3$ + 12.

    (b) Determine where the graph of the f(x) = x^2 + $1/x^2$ is concave downward and concave upward.

19. Use the definition of area, find the area of the region under the graph of $f(x) = 3x - 1,$ on $[1, 3],$ by choosing C_k as the mid point.

20. Suppose that f is continuous on $[-a, a]$. Then show that

    (a) If f is even, then $\int_{-a}^{a} f(x)dx$= 2 $\int_0^a f(x)dx$

    (b) If f is odd, then $\int_{-a}^{a} f(x)dx$ = 0.

21. Find the area of the region bounded by the graphs of $x = y^2$ and $y = x - 2$

22. Find the volume of the solid obtained by revolving the region under the graph of $x = \frac{1}{y}, x = 0, y = 1$ and $y = 2$ about the y- axis

23. Find the area of the surface obtained by revolving the graph of $y = 4 - x^2$ on the interval $[0, 2]$ about the y-axis

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. (a) State and prove the First Derivative Test.

    (b) Explain the procedure to find the relative extrema of a continuous function using first derivative test.

    (c) Find the relative extrema of f(x) = x^4 - $4x^3$ + 12

25. Sketch the graph of the function f(x) = $2x^3$ - $9x^2$ + 12x - 3.

26. State and prove both Part 1 and Part 2 of Fundamental Theorem of Calculus.

27. (a) Find the length of the graph $f(x) = \frac{1}{3}x^3 + \frac{1}{4x}$ on the interval [1,3]

    (b) Find the length of the graph $x = \frac{1}{3}y^3 + \frac{1}{4y}$ from y=1 to y=2

(2 × 10 = 20 Marks)

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