

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS2 C02 / CC20U MTS2 C02 - MATHEMATICS - II

(Mathematics - Complementary Course)

(2019 Admission onwards)

Time : 2.00 Hours

Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)Answer *all* questions. Each question carries 2 marks.

1. Convert $(8, \frac{3\pi}{2})$ from polar coordinates to Cartesian coordinates.
2. Show that $f(x) = \frac{1}{3}x^3 - x$ is not invertible on any of the interval containing 1.
3. Replace the polar equation $r = 1$ by equivalent Cartesian equation. Then identify the graph.
4. Evaluate $\int \frac{\sinh x}{\cosh^4 x} dx$.
5. Express $\sinh^{-1}(5)$ in terms of natural logarithms.
6. Find $\lim_{n \rightarrow \infty} \left(\frac{n^2 + 1}{3n^2 + n} \right)$.
7. Show that the series $\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \dots$ diverges.
8. Sum the series $\sum_{i=1}^{\infty} \frac{2^{3i+4}}{3^{2i+5}}$.
9. Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ converges. Find its sum with an error of no more than 0.04.
10. Determine whether the set given by $\{f \in C(-\infty, \infty) : f(0) = 5\}$ is a subspace of $C(-\infty, \infty)$? Justify.
11. If $A = \begin{bmatrix} 0 & 2 & 4 & 0 \\ 1 & 2 & -2 & 3 \\ 5 & 1 & 0 & -1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$. Evaluate M_{41} .
12. Find the inverse of the matrix $\begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix}$.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer **all** questions. Each question carries 5 marks.

13. Find the length of the curve $f(x) = x^{3/2}$ on $[0, 4]$.
14. Determine the convergence or divergence of $\int_2^{\infty} \frac{1}{x \ln x} dx$.
15. Evaluate $\int_0^{\pi/2} \cos x dx$ using trapezoidal rule with $n = 10$. Compare the answer with the true value.
16. Let $B = \{w_1, w_2, w_3\}$ where $w_1 = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$, $w_2 = \langle \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$, $w_3 = \langle 0, \frac{1}{\sqrt{2}}, 1 \frac{1}{\sqrt{3}} \rangle$. Show that B is an orthonormal basis. Express $u = \langle 3, -2, 9 \rangle$ in terms of B.
17. Find the rank of the matrix $\begin{bmatrix} 1 & -2 & 1 & 8 & -1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 3 & -1 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 & -1 & 2 & 10 & 8 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 3 \\ 1 & -2 & 1 & 8 & -1 & 1 & 2 & 6 \end{bmatrix}$
18. Find the eigen values and the corresponding eigen vectors of the matrix $A = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$. Then without finding A^{-1} find its eigen values and corresponding eigen vectors.
19. Compute A^m if $A = \begin{pmatrix} 5 & -3 \\ -3 & 5 \end{pmatrix}$. Hence find A^4

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any **one** question. The question carries 10 marks.

20. Balance the chemical equation $Na + H_2O \rightarrow NaOH + H_2$.
21. Diagonalize the matrix $\begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$.

(1 × 10 = 10 Marks)
