22U401 (Pages: 2) Name: ....

Reg.No: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS4 B04 / CC20U MTS4 B04 - LINEAR ALGEBRA

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Find the parametric equations corresponding to the solution set of linear equation $3x - 5y + 4z = 7$
2. Give the inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
3. Prove that, elementary matrix is invertible and the inverse is also an elementary matrix.
4. Find A^{-1}, if $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
5. Find the Values of λ for which $det(A) = 0$, when $A = \begin{bmatrix} \lambda - 1 & 0 \\ 2 & \lambda + 1 \end{bmatrix}$
6. Define Linearly independent set.
7. Find the coordinate vector of$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ relative to the standard basis of $M_{2\times 2}$
8. Prove that, if $W$is a subspace of a finite dimensional vector space V, then W is finite dimensional.
9. Define transition matrix.
10. Define Row vectors.
11. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $w_1 = 2x_1 + x_2$ and $w_2 = 3x_1 + 4x_2$, check whether T(1,1)=T(1,0)+T(0,1)
12. Find the equation of the image of the line $y = 4x$ under the multiplication by matrix $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$
13. Define Eigen Value.
14. When we say 2 vectors are orthogonal in an inner product space?
15. When we say that the matrix B is orthogonally similar to A.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Prove that a system of linear equations has zero,one or infinitely many solutions.

17. What conditions must b_1, b_2 and b_3 satisfy in order for the system of equations $x_1 + x_2 + 2x_3 = b_1; x_1 + x_3 = b_2; 2x_1 + x_2 + 3x_3 = b_3$ be consistent.

18. Let $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation.Find the standard matrix for the transformation and find $T(x), if\ T(e_1) = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $T(e_2) = \begin{bmatrix} -3 \\ -1 \\ 0 \end{bmatrix}$, $T(e_3) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $X = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

19. Show that the solution of a homogeneous linear system $Ax = 0$ in n unknowns is a subspace of $\mathbb{R}^n$

20. Use matrix multiplication to find the reflection of $(1, -3, 2)$ about xy- plane, xz plane and the yz plane.

21. Find a matrix P that diagonalize $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$

22. Show that $\langle f, g \rangle = \int_a^b f(x).\, g(x)$ is an inner product space on $C[a, b]$

23. Let $S = \{v_1, v_2, v_3\}$ where $v_1 = (0, 1, 0), v_2 = (\frac{-4}{5}, 0, \frac{3}{5}), v_3 = (\frac{3}{5}, 0, \frac{4}{5})$ express $u = (1, 1, 1)$ as a linear combination of vectors in S.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. (a) Prove that $\begin{vmatrix} a_1 + b_1 t & a_2 + b_2 t & a_3 + b_3 t \\ a_1 t + b_1 & a_2 t + b_2 & a_3 t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (1 - t^2) \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

    b) If B is the matrix that results when a single row or single column of A is multiplied by a scalar k then prove that $det(B) = k.\, det(A)$

25. Show that the spaces $R^2\ and\ R^3$ are vector spaces.

26. a) Prove that if A is an $m \times n$ matrix and if $m > n$, then the linear system $Ax = b$ is inconsistent for atleast one vector b in $\mathbb{R}^n$

    b) Prove that if A is an $m \times n$ matrix and if $m > n$, then for each vector b in $\mathbb{R}^n$ the linear system $Ax = b$ is either inconsistent or has infinitely many solutions

27. Prove that the following are equivalent for an $n \times n$ matrix A.

    (a) A is orthogonal

    (b) The row vectors of A form an orthonormal set in $\mathbb{R}^n$ with the Euclidean inner product.

    (c) The Column vectors of A form an orthonormal set in $\mathbb{R}^n$ with the Euclidean inner product.

(2 × 10 = 20 Marks)

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