21U602 (Pages: 2) Name: ....

Reg. No: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS6 B11 / CC20U MTS6 B11 - COMPLEX ANALYSIS

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 5

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Evaluate $\lim_{z \to -i} \left(\frac{z^4 - 1}{z + i} \right)$
2. Show that $f(z) = z^2 - iz + 3 - 2i$ is continuous at $z_0 = 2 - i$
3. If $f(z) = \frac{3e^{2z} - ie^{-z}}{z^3 - 1 + i}$, find the derivative $f'(z)$.
4. Show that the function $f(z) = \frac{\cos\theta}{r} - i\frac{\sin\theta}{r}$ is analytic in an appropriate domain.
5. Prove that $\frac{d}{dz}(e^z) = e^z$.
6. Find $\frac{d}{dz}(\sin z^2)$.
7. Show that $\sinh(iz) = i \sin z$.
8. Evaluate $\int_C (3x^2 + 6y^2) ds$ where the path of integration C is given by $y = 2x + 1;\ -1 \leq x \leq 0$.
9. Without evaluating find an upper bound for the absolute value of the integral $\int_C (z^2 + 4) dz$ where C is the line segment from $z = 0$ to $z = 1 + i$.
10. Define the terms, (a) Simply connected domains (b) Multiply connected domains.
11. Prove that the bounded entire functions are constants.
12. Determine whether the sequence $\left\{ \frac{4n + 3ni}{2n + i} \right\}$ converges or diverges.
13. Discuss the convergence of the series $\sum_{k=1}^{\infty} \frac{ik + 5}{k}$
14. Determine the zeros and their order for the function $f(z) = z^4 + z^2$

15. Let $f(z) = \dfrac{1}{(z-1)^2(z-3)}$, find $\text{Res}(f(z), 3)$.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Suppose $f(z)$ is analytic. Can $g(z) = \overline{f(z)}$ analytic? Explain.

17. Find all complex solutions of the equation $e^w = 1 + i$.

18. State and prove the fundamental theorem for contour integrals.

19. State Cauchy's integral formula. Using Cauchy's integral formula evaluate $\oint_C \dfrac{z^2+4}{z^2-5iz-4} dz$ where C is the circle $|z-3i| = 1.3$.

20. Evaluate $\oint_C \dfrac{1}{z^3(z-1)^2} dz$ where C is the circle $|z-2| = 5$.

21. Find the circle and radius of convergence of the power series $\sum_{k=1}^{\infty} \dfrac{z^{k+1}}{k}$

22. Expand $f(z) = \dfrac{1}{3-z}$ in a Taylor series with center $z_0 = 2i$. Give the radius of convergence R.

23. Evaluate $\int_0^{2\pi} \dfrac{1}{10 - 6\cos\theta} d\theta$

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Show that the function $u(x,y) = e^x(x\cos y - y\sin y)$ is harmonic in the entire complex plane. Also find the harmonic conjugate function of u.

25. Evaluate $\int_C \text{Im}(z-i)dz$ where C is the polygonal path consisting of the circular arc along $|z| = 1$ from $z = 1$ to $z = i$ and the line segment from $z = i$ to $z = -1$.

26. Expand $f(z) = \dfrac{1}{z(z-3)}$ in a Laurent series valid for the following annular domains

$0 < |z| < 3$

$|z| > 3$

$0 < |z-3| < 3$

$|z-1| > 3$

27. State residue theorem. Using residue theorem evaluate $\oint_C \dfrac{1}{(z-1)^2(z-3)} dz$ where C is

(i) Rectangle defined by $x = 0,\ x = 4,\ y = -1,\ y = 1,$ (ii) Circle $|z| = 2$.

(2 × 10 = 20 Marks)

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