21U603 (Pages: 2) Name: ....

Reg.No: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS6 B12 / CC20U MTS6 B12 - CALCULUS OF MULTI VARIABLE

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Define level curves of a function of two variables.
2. Find the first partial derivatives of the function $f(x,y) = x\sqrt{y}$.
3. Find $\dfrac{dy}{dx}$ if $x^3 + xy + y^2 = 4$.
4. Suppose f is a differentiable at the point (x,y). Then prove that the maximum value of $D_u f(x,y)$ is $|\bigtriangledown f(x,y)|$, and this occurs when u has the same direction as $\bigtriangledown f(x,y)$.
5. Show that the point $(0,0)$ is a critical point of $f(x,y) = y^2 - x^2$.
6. Evaluate$\int_1^2 \int_0^1 3x^2 ydxdy$.
7. Evaluate $\int\int_r (1+2x+2y)dA$, where$R = \{(x,y) : 0 \leq y \leq 1, y \leq x \leq 2y\}$.
8. Evaluate $\int\int_R xdA$ where R is the disk of radius 1 centered at the origin.
9. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^3 \int_0^r rdzdrd\theta$
10. Derive the formula for changing double integral in rectangle coordinates to polar coordinates.
11. Find the gradient vector field of $f(x,y) = x^2y - y^3$.
12. Find the $curlF(-1,2,1)$, if $F(x,y,z) = xy\hat{i} + xz\hat{j} + xyz^2\hat{k}$.
13. Find a parametric representation for the cone $x = \sqrt{y^2+z^2}$.
14. State divergence theorem.
15. State Stockes' Theorem.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Find the second-order partial derivatives of $f(x,y) = x^4 - 2x^2y^3 + y^4 - 3x$.

17. Let $z = 2x^2 - xy$.
    (i) Find the differential dz.
    (ii)Compute the value of dz if (x,y)changes from (1,1) to (0.98,1.03).

18. Find equations of the tangent plane and normal line to the surface with equation$x^2 + 4y^2 + 9z^2 = 17$ at the point $(2,1,1)$.

19. Find the surface area of the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$.

20. Evaluate$\int\int\int_T z dV$ where T is the solid in the first octant bounded by the graphs of $z = 1 - x^2$ and $y = x$.

21. Let $F(x,y) = -\frac{1}{8}(x-y)\hat{i} - \frac{1}{8}(x+y)\hat{j}$ represents a force field. Find the work done on a particle that moves along the quarter-circle of radius 1 centered at the origin in a clockwise direction from $(0,1)$ to $(1,0)$.

22. Using Green's theorem, evaluate $\oint_C (y^2 + \tan x)dx + (x^3 + 2xy + \sqrt{y})dy$, where C is the circle $x^2 + y^2 = 4$ and is oriented in a positive direction.

23. Find the mass of the surface S composed of the part of the paraboloid $y = x^2 + z^2$ between the planes $y = 1$ and $y = 4$ if the density at a point P on S is inversely proportional to the distance between P and the axis of symmetry of S.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. (i) Show that $lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.
    (ii) Using $\varepsilon - \delta$ definition of limits, prove that $lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+y^2} = 0$.

25. Find the absolute extreme values of $f(x,y) = 2x^2 + y^2 - 2y + 1$ subject to the constraint $x^2 + y^2 \leq 4$.

26. Find the center of mass of the region bounded by the graph $y = \sqrt{x}, y = 0, x = 4, \rho(x,y) = xy$.

27. Let $F(x,y,z) = yz^2\hat{i} + xz^2\hat{j} + 2xyz\hat{k}$.
    (a) Show that F is conservative, and find a potential function f such that $F = \bigtriangledown f$.
    (b) If F is a force field, find the work done by F in moving a particle along any path from $(0,0,1)$ to $(1,3,2)$.

(2 × 10 = 20 Marks)

****