

21U604

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Name:

Reg.No:

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC20U MTS6 B13 - DIFFERENTIAL EQUATIONS

(Mathematics - Core Course)

(2020 Admission onwards)

Time : 2.5 Hours

Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer *all* questions. Each question carries 2 marks.

1. Solve the initial value problem $\frac{dy}{dt} = -2y + 10; y(0) = 0$
2. Determine the order of the given differential equation $\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + 2y = \sin t$. Also state whether the equation is linear or nonlinear.
3. Determine the value of r for which the differential equation $y'' + 5y' + 6y = 0$ has solutions of the form $y = e^{rt}$.
4. Find an interval in which the initial value problem $ty' + 2y = 4t^2; y(1) = 2$ has a unique solution.
5. Find the general solution of the $y'' + 5y' + 6y = 0$
6. Find the Wronskian of $y_1 = \sin t, y_2 = \cos t$. Determine whether y_1 and y_2 are linearly independent.
7. Find the general solution of $y'' - 2y' + 2y = 0$
8. Write the particular solution of $y'' - 2y' - 3y = 2e^{4t}$
9. Write the formula for finding the particular solution of the differential equation by using the method of variation of parameters.
10. Find the Laplace transform of $\cos at$
11. Find the inverse Laplace transform of $\frac{n!}{(s-a)^{n+1}}$
12. Write the expression for $\mathcal{L}(f'(t))$ and $\mathcal{L}(f''t)$
13. Show by an example that $f * f$ is not necessarily nonnegative
14. Solve the boundary value problem $y'' + y = 0; y(0) = 1, y(\pi) = -1$

15. Prove that the product of an even function and an odd function is odd.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer *all* questions. Each question carries 5 marks.

16. Solve the initial value problem $\frac{dy}{dx} = \frac{1 - 2x}{y}$; $y(1) = -2$

17. Find an integrating factor of $(3x^2y + 2xy + y^3) + (x^2 + y^2)y' = 0$

18. Find a second independent solution of $t^2y'' + 3ty' + y = 0$, $t > 0$; $y_1(t) = t^{-1}$

19. Find a series solution in powers of $(x - 1)$ of the Airy's equation $y'' - xy = 0$, $-\infty < x < \infty$

20. Find the inverse Laplace transform of $\frac{2(s - 1)e^{-2s}}{s^2 - 2s + 2}$

21. Find the Laplace transform of $f(t) = \int_0^t \sin(t - \tau) \cos(\tau) d\tau$

22. Let $f(x) = \begin{cases} 0 & -3 < x < -1, \\ 1, & -1 < x < 1, \\ 0 & 1 < x < 3. \end{cases}$ and suppose that $f(x + 6) = f(x)$. Find the coefficients in the Fourier series for $f(x)$.

23. Let $f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1, & 0 \leq x < \pi \end{cases}$ $f(x + 2\pi) = f(x)$. Find the Fourier series of $f(x)$.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any *two* questions. Each question carries 10 marks.

24. Solve the differential equation $(y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$

25. Find the solution of the initial value problem $y'' + 4y = 3 \sin 2t$, $y(0) = 2$, $y'(0) = -1$

26. Using Laplace transform solve the initial value problem $y'' + 2y' + 5y = 0$; $y(0) = 2$, $y'(0) = -1$

27. Find the temperature $u(x, t)$ at any time in a metal rod 40cm long, insulated on the sides with $u(x, 0) = x$ and whose ends are maintained at $0^\circ C$ for all $t > 0$.

(2 × 10 = 20 Marks)
