C 83649 (Pages : 3) Name..............................

26

Reg. No..............................

SECOND SEMESTER M.Sc. DEGREE (CUCSS) EXAMINATION
JUNE 2015

Statistics

ST 2C 06—ESTIMATION THEORY

(2013 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Weightage 1 *for each question.*

1. Let $X_1, X_2, \ldots, X_n$ be random sample of size n from $B(\alpha, \beta)$, find a sufficient statistic for α when β is known.
2. Define complete sufficient statistic.
3. State Lehmann-Scheffe theorem.
4. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ obtain an ancilliary statistic for σ^2.
5. Define exponential family of distributions. Give an example.
6. What is meant by a CAN estimator ?
7. State the application of Fisher Neymann factorization criterion.
8. Explain method of percentile estimation.
9. Define one parameter Cramer family. Give an example.
10. Describe method of moment estimation for finding consistent estimator.
11. Define UMA unbiased confindence interval.
12. Distinguish between confidence interval and fiducial interval.

(12 × 1 = 12 weightage)

Part B

Answer any **eight** *questions.*
Weightage 2 *for each question.*

13. Let X_1 be a Bernonulli random variable with $P[X_1 = 1] = p$ and $P[X_1 = 0] = 1 - p$ and let X_2 be another Bernoulli random variable with $P[X_2 = 1] = 2p$ and $P[X_2 = 0] = 1 - 2p$, $0 < p < 1/2$ and X_1 and X_2 are independent. Show that $X_1 + X_2$ is not sufficient for p.

Turn over

14. Explain the procedure to obtain the UMVU estimator in the presence of a complete sufficie statistics.

15. State and prove Cramer-Rao inequality for the multiparameter case.

16. State and prove Basu's theorem.

17. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from a Poisson distribution with mean θ find the UMVU of $P(X_1 < 1)$.

18. Give an example where the Cramer-Rao lower bound is attained and another where it is n attained.

19. State the optimum properties of MLE and prove any one of them.

20. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from the distribution having p.d.f.

$$f(x, \sigma) = \begin{Bmatrix} \frac{1}{\sigma} e^{-\left(\frac{x}{\sigma}\right)}, \text{ for } 0 < x < \infty \,;\, 0 < \sigma < \infty \\ 0, \text{ elsewhere.} \end{Bmatrix}.$$

Find the MLE of σ and show that it is consistent and asymptotically normal.

21. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from $B(\alpha, \beta)$. Find the method of moments estimator (α, β).

22. Define shortest length confidence interval and explain the role of sufficient statistic in determ the same.

23. Obtain the confidence interval for σ^2 based on a random sample $X_1, X_2, \ldots\ldots, X_n$ form $N(\mu, \sigma$ when μ is known.

24. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from $U(0, \theta)$. Find the unbiased confidence interval θ based on the pivot $\frac{\text{Max } X_i}{\theta}$.

(8 × 2 = 16 weighta

Part C

Answer any **two** *questions.*
Weightage 4 for each question.

25. (a) Prove or disprove "A complete sufficient statistic is minimal sufficient".

(b) State and prove Rao-Blackwell theorem.

26. (a) Find a consistent estimator of the parameter θ of the distribution with p.d.f.

$$f(x, \theta) = \frac{1}{\pi} \frac{1}{1 + (x - \theta)^2}, -\infty < x < \infty, -\infty < \theta < \infty.$$

(b) Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations from $N(\mu, \sigma^2)$ obtain CAN estimators of (μ, σ).

27. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \mu^2)$, find the MLE of μ.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from uniform $U(0, \theta)$ distribution. For estimating θ using the squared error loss function, a prior density of θ is given by

$$\pi(\theta) = \frac{\alpha a^\alpha}{\theta^{\alpha+1}}, \theta \geq a.$$

Find the Bayes estimator of θ.

28. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from $G(1, \theta)$. Find the unbiased confidence interval for θ with confidence level $1 - \alpha$ based on the pivot $2\sum_{i=1}^{n} X_i / \theta$.

(b) Find a confidence interval with confidence coefficient α for the difference of means of two normal populations with common unknown variances.

(2 × 4 = 8 weightage)