C 83623 (Pages : 3) Name..................................

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SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2015

(CUCSS)

Mathematics

MT 2C 06—ALGEBRA—II

Time : Three Hours Maximum : 36 Weightage

Part A

Short answer questions (1-14).

Answer **all** *questions. Each question has* 1 *weightage.*

1. Is $\mathbb{Q}[x]/\langle x^2-2\rangle$ a field ? Justify your answer.
2. Show that the polynomial x^2+1 is irreducible in $\mathbb{Z}_3[x]$.
3. Is $\mathbb{C}$ a simple extension over $\mathbb{R}$? Justify your answer.
4. Find $\left[\mathbb{Q}\left(\sqrt[3]{2}\right):\mathbb{Q}\right]$.
5. Let E be a finite extension of degree n over a finite field F. If F has q elements, then prove that E has q^n elements.
6. Does there exist a field of 4096 elements ? Justify your answer.
7. Prove that a finite extension E of a finite field F is a simple extension of F.
8. Find all conjugates of $3+\sqrt{2}$ over $\mathbb{Q}$.
9. Find the splitting field of $\{x^2-2, x^3-3\}$ over $\mathbb{Q}$.
10. Show that $\mathbb{Q}\left(\sqrt[3]{2}\right)$ has only the identify automorphism.
11. Define separable extension of a field. Give a separable extension of $\mathbb{Q}$.
12. Let p be a prime, $\mathrm{F}=\mathbb{Z}_p$ and let $\mathrm{K}=\mathrm{GF}\left(p^{12}\right)$. Find $\mathrm{G}(\mathrm{K/F})$.
13. Is regular 7-gon constructible ? Justify your answer.
14. Prove that the polynomial x^5-1 is solvable by radicals over $\mathbb{Q}$.

(14 × 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *from the following ten questions* (15-24).
Each question has weightage 2.

15. Let F be a field. Prove that every ideal in $F[x]$ is a principal ideal.

16. Prove that a finite extension is an algebraic extension.

17. Prove that $\mathbb{Q}\left(\sqrt{3}+\sqrt{7}\right)=\mathbb{Q}\left(\sqrt{3},\sqrt{7}\right)$.

18. Show that if E is a finite extension of a field F and $[E:F]$ is a prime number, then E is a sir

    extension of F and $E=F(\alpha)$ for every $\alpha\in E$ with $\alpha\notin F$.

19. Let F be a field and let $f(x)$ be irreducible in $F[x]$. Prove that all zeros of $f(x)$ in $\overline{F}$ have

    same multiplicity.

20. Let F be a subfield of a field E. Prove that the set all automorphisms of E leaving F fixed for
    subgroup of the group of all automorphisms of E.

21. Show that if $[E:F]=2$, then E is a splitting field over F.

22. Let K be a finite extension of E and E be a finite extension of F. Prove that K is separable o
    if and only if K separable over E and E is separable over F.

23. Describe the group of polynomial $\left(x^3-1\right)\in\mathbb{Q}[x]$ over $\mathbb{Q}$.

24. Prove that the Galois group of the pth cyclotonic extension of $\mathbb{Q}$ for a prime p is cyclic of

    $p-1$.

(7 × 2 = 14 weigh

Part C

Answer any **two** *from the following four questions* (25-28).
Each question has weightage 4.

25. (a) Prove that x^2-3 is irreducible over $\mathbb{Q}\left(\sqrt[3]{2}\right)$.

    (b) Let E be a finite extension field of a field F and let K be a finite extension field of E. Prov
    K is a finite extension field of E and

$$[K:F]=[K:E][E:F].$$

26. Let F be a field of characteristic p. Prove that the map $\sigma_p : F \to F$ defined by $\sigma_p(a) = a^p$ is an automorphism. Also, prove that $F_{\{\sigma_p\}} = \mathbb{Z}_p$.

27. Prove that every finite field is perfect.

28. Let F be a field of characteristic zero and let $F \leq E \leq K \leq \overline{F}$, where E is a normal extension of F and K is an extension of F by radicals. Prove that $G(E/F)$ is a solvable group.

(2 × 4 = 8 weightage)