15P201 (Pages: 2)

Name…………………

Reg.No………………

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JULY 2016

(CUCSS - PG)

(Mathematics)

CC15PMT2C06-ALGEBRA-II

(2015 Admission)

Time: 3 Hours Maximum: 36 Weightage

Part A

Answer **all** questions

Each question carries 1 weightage.

1. Find all prime ideals and all maximal ideals of Z_6.
2. Show that the polynomial $x^3 - 2$ has no zeroes in $Q(\sqrt{2})$
3. Find the degree and basis of $Q(\sqrt{2}, \sqrt{3}, \sqrt{18})$ over Q.
4. Prove that squaring the circle is impossible.
5. Find the primitive 5 th root of unity in Z_{11}.
6. State Conjugation Isomorphism theorem.
7. Find all conjugates in C of $3 + \sqrt{2}$ over Q.
8. Find $\{Q(\sqrt{2}, \sqrt{3}): Q\}$
9. State isomorphism extension theorem.
10. Let K be a finite normal extension of F and let E be an extension of F, where $F \leq E \leq K \leq \bar{F}$. Prove that K is a finite normal extension of E.
11. Describe the group of the polynomial of $x^4 - 1$ over Q.
12. Find $\emptyset_6(x)$ over Q.
13. Show that the polynomial $x^5 - 1$is solvable by radicals over Q.
14. Determine whether there exist a finite field having 4096 number of elements.

(14×1 = 14 weightage)

Part B

Answer **any seven** questions

Each question carries 2 weightage.

15. Prove that if F is a field, every ideal in F[x] is principal.
16. Let R be a finite commutative ring with unity. Show that every prime ideal in R is a maximal ideal.
17. If E is a finite extension field of a field F and K is a finite extension field of E, Prove that K is a finite extension of F and $[K:F] = [K:E][E:F]$.
18. Prove that a finite field $GF(p^n)$ of p^n elements exists for every prime power p^n.
19. Find a basis for $Q(2^{1/2}, 2^{1/3})$ over Q.

20. Prove that complex zeros of polynomials with real coefficients occur in conjugate pairs.

21. Let $\bar{F}$ and $\bar{F}'$ be two algebraic closures of F. Prove that $\bar{F}$ is isomorphic to $\bar{F}'$ under an isomorphism leaving each element of F fixed.

22. State main theorem of Galois Theory.

23. Prove that the Galois group of the n^{th} cyclotomic extension of Q has $\emptyset(n)$ elements and is isomorphic to the group consisting of the positive integers less than n and relatively prime to n under multiplication modulo n.

24. Let F be a field of characteristic zero, and let $\leq E \leq K \leq \bar{F}$, where E is a normal extension of F and K is an extension of F by radicals. Prove that $G\ (E\ /F)$ is a solvable group.

(7×2 = 14 weightage

Part C

Answer any two questions

Each question carries 4 weightage.

25. Let F be a field and let f(x) be a non-constant polynomial in F[x]. Prove that there exist an extension field E of F and $\alpha \in E$ such that $f(\alpha) = 0$

26. Prove that field E, where $F \leq E \leq \bar{F}$ is a splitting field over F if and only if every automorphism of F^{-}leaving F fixed maps E onto itself and thus induces an automorphism of E leaving F fixed.

27. Prove that the regular n – gon is constructable with a compass and a straightedge if and only if all the odd primes dividing n are Fermat primes whose squares do not divide n.

28. Prove that every finite field is perfect.

(2×4 = 8 weightag

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