15P253 (Pages: 2) Name.......................

Reg. No....................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JULY 2016

(CUCSS - PG)

(Statistics)

CC15P ST2 C06 - ESTIMATION THEORY

(2015 Admission)

Time: Three Hours Maximum: 36 Weightage

PART A

(Answer all questions. Weightage **1** *for each question)*

1. Define Minimal sufficient statistic.
2. Give an example to prove that MLE's are not unbiased.
3. Explain Fisher information.
4. Define Bayesian estimation.
5. Define consistency. Let $X \sim U(0, \theta)$. Show that $X_{(n)} = \max(X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ is consistent.
6. Define UMVUE.
7. Describe the method of construction of confidence intervals using pivots.
8. Define Best Linear Unbiased Estimator.
9. Explain the method of percentiles for estimation of parameters.
10. Find the Cramer-Rao lower bound of the variance of the unbiased estimator of θ with $f(x;\theta) = \theta(1-\theta)^x, x = 0,1,2 \ldots$ and $0 < \theta < 1$.
11. Let $X \sim U(\theta, \theta+1)$, Find sufficient statistic for θ.
12. Define one parameter exponential family of distributions.

(12*1=12 weightage)

PART B

(Answer any **eight** *questions. Weightage* **2** *for each question)*

13. State and prove Neyman-Factorization theorem.
14. Define complete family of distributions. Give an example.
15. Define MLE. Prove or disprove: MLE's are always consistent.
16. State and Prove Lehmann-Scheffe theorem.
17. State and prove Cramer-Rao inequality.

18. Prove or disprove: " If T_n is a CAN estimator of θ then ${T_n}^k$ is a CAN estimator of θ^k, k is a known positive integer".
19. State and prove invariance property of consistent estimator.
20. Let $X_1, X_2, \dots, X_n$ be a random sample of size drawn from a Poisson distribution with parameter $\boldsymbol{\lambda}$.Check the consistency and unbiasedness of the estimator $\boldsymbol{T} = \frac{2}{n(n+1)} \sum_{i=1}^{n} \boldsymbol{X_i}$ of $\boldsymbol{\lambda}$.
21. Explain the method of construction of confidence interval using maximum likelihood estimator. Illustrate with an example.
22. State and prove invariance property of MLE.
23. Distinguish between Bayesian and Fiducial interval.
24. Let $X \sim P(\lambda)$. Find a BLUE for λ.

PART C

(Answer **two** *questions. Weightage* **4** *for each question)*

25. Apply method of moment estimation to estimate the parameter θ of the following distribution with pdf.

$$f(x;\theta) = \frac{1}{2\theta} e^{\frac{-|x|}{\theta}}, -\infty < x < \infty, \theta > 0.$$

Verify the obtained estimator is CAN estimator.

26. Let $X_1, X_2, \dots, X_n$ be a random sample of size drawn from a Poisson distribution with parameter $\boldsymbol{\lambda}$. Assuming that the prior distribution of λ is $G(\alpha, \beta)$. Find $100(1-\alpha)\%$ Bayesian confidence interval for λ. Compare it with classical shortest length confidence interval.
27. a) Explain Cramer family.

    b) State and prove Cramer-Huzurbazar theorem.
28. State and Prove Rao-blackwell theorem

****