15P204 (Pages: 3) Name……………………..

Reg. No………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JULY 2016

(CUCSS - PG)

(Mathematics)

CC 15P MT2 C09 - PDE & INTEGRAL EQUATIONS

(2015 Admission)

Time: Three Hours Maximum: 36 Weightage

Part A

(Answer all questions, each question has 1 weightage)

1. Find the partial differential equation by eliminating the arbitrary function F from the equation $z = x + y + F(xy)$.
2. Show that $z = ax + by + a^2 + b^2$ is a complete integral of $z - px - qy - p^2 - q^2 = 0$.
3. Determine the region D in which the two equations $p^2 + q^2 - 1 = 0$ and $(p^2 + q^2)x - pz = 0$ are compatible.
4. What is characteristic strip?
5. Find the complete integral of $p + q - pq = 0$.
6. Write the classification of the equation : $u_{xx} + 2u_{xy} + 17u_{yy} = 0$
7. What is the Neumann problem for the upper half plane.
8. State Harnack's theorem.
9. What is Riemann function?
10. What is the Dirichlet problem for the upper half plane.
11. Differentiate between Fredholm and Voltara integral equations.
12. If $y''(x) = F(x)$ and y satisfies the conditions $y(0) = 0$ and $y(1) = 0$, show that $y(x) = \int_0^1 K(x,\xi)F(\xi)d\xi$, where $K(x, \xi) = \begin{cases} \xi(x-1), x > \xi \\ x(\xi-1), x < \xi \end{cases}$
13. Determine the resolvent kernel associated with $K(x, \xi) = x\xi$ in (0, 1) in the form of power series in λ obtaining the first three terms.
14. Prove that the characteristic numbers of a Fredholm equation with real symmetric kernel are all real.

Part B

(Answer any seven from the following ten questions (15-24),

each question has 2 weightage)

15. Find the general integral of the equation $(z^2 - 2yz - y^2)p + x(y + z)q = x(y - z)$.
16. Find the complete integral of the equation $x^2 p^2 + y^2 q^2 - 4 = 0$ by Charpit's method.
17. Solve the equation $p^2x + q^2y = z$ by Jacobi's method .
18. Find the integral surface of the equation $x^3p + y(3x^2 + y)q = z(2x^2 + y)$ which pass through $x =1, y = s, z = s(1 + s)$.
19. Reduce the equation $x^2 u_{xx} - y^2u_{yy} = 0$ into canonical form.
20. Obtain D'Alembert's solution of one dimensional wave equation which describes tl vibrations of an infinite string.
21. Prove that the solution $u(x, t)$ of the differential equation $u_t - ku_{xx} = F(x, t), 0 < x < l, \ t > 0$ satisfying the initial conditions $u(x, 0) = f(x), 0 \leq$ and the boundary conditions $u(0, t) = u(l, t) = 0, t \geq 0$ is unique
22. Transform the problem $y'' + y = x; y(0) = 0, y'(1) = 0$ to a Fredholm integral equatic using Green's function.
23. Solve the Fredholm integral equation by iterative method : $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi$.
24. Determine the iterated kernel $K_2(x, \xi)$ associated with $K(x, \xi) = |x - \xi|$ in (0, 1).

Part C

(Answer any two from the following four questions (25-28),

each question has 4 weightage)

25. Show that a necessary and sufficient condition that the pfaffian differential equation $\vec{X}.d\vec{r} = P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$ be integrable is that $\overrightarrow{X}.curl\ \vec{X}$
26. Using the method of characteristics, find an integral surface of $pq = xy$ which passe through the line $x = z, y = 0$.
27. (a) Show that $v(x, y; \alpha, \beta) = J_0 \sqrt{(x - \alpha)(y - \beta)}$ is the Riemann function for the second order partial differential equation $u_{xy} + \frac{1}{4} u = 0$, where J_0 denotes the Bessel's function of first kind of order zero.

    (b) Solve the Dirichlet problem for a circle by choosing a suitable Green's functior

28. (a) Consider the equation $y(x) = 1 + \lambda \int_0^{2\pi} \sin(x + \xi)\, y(\xi)\, d\xi$.

Determine the characteristic values of λ and the corresponding characteristic functions.

(b) Obtain the most general solution of the equation

$y(x) = \lambda \int_0^{2\pi} \sin(x + \xi)\, y(\xi)\, d\xi + x$ under the assumption that $\lambda \neq \pm \frac{1}{\pi}$.

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