15P255 (Pages: 2) Name…………………

Reg. No…………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JULY 2016

(CUCSS - PG)

(Statistics)

CC15P ST2 C08 - PROBABILITY THEORY

(2015 Admission)

Time: Three Hours Maximum: 36 Weightage

Part A Answer *all* questions

1. Define Distribution function of a random variable.
2. What is meant by independence of Random variables?
3. State Kolmogorov 0-1 Law.
4. Define Convergence almost sure.
5. Let $P[X_n = 0] = 1 - n^{-r}, P[X_n = n] = n^{-r}, r \geq 2, n = 1,2,$...Show that $X_n \xrightarrow{a.s} 0$ but X_n does not converge to zero in r^{th} mean.
6. Define Kolmogorov's WLLN's.
7. Define submartingale and supermartingale.
8. Define convergence in r^{th} mean.
9. Define characteristic function. Give two properties
10. State continuity theorem. Give application.
11. What is inversion theorem?
12. State Lindberg-Feller Central Limit theorem

(12*1=12 weightage)

Part B Answer any *eight* questions

13. Show that conditional probability is a particular case of conditional expectation.
14. If $X_n \xrightarrow{P} X$ and g is a continuous real valued function, then show that $g(X_n) \xrightarrow{P} g(X)$.
15. Does the WLLN's hold for the following sequence of independent random variables $P[X_n = \pm 1] = \frac{1}{2}$.
16. What do you mean by convergence in distribution?
    Examine the convergence of

    $F_n(x) = 0$, if $x < n$

    $= 1$, if $x \geq n$.
17. If $X_n \xrightarrow{a.s} X$, show that $X_n \xrightarrow{P} X$.
18. State and prove Borell Cantelli Lemma
19. Let $\{X_n\}$ be a sequence of i.i.d. random variables with common mean μ. Then Show that, $\frac{S_n}{n} \to \mu$ in probability as $n \to \infty$.
20. State and prove Kolmogrov's three series criterion.

21. Let { X_n, Y_n } be a sequence of pairs of random variables with $X_n \xrightarrow{L} X$ and $Y_n \xrightarrow{P} c$. Show that $X_n . Y_n \xrightarrow{L} cX$, if $c \neq 0$ and $X_n Y_n \xrightarrow{p} 0$, if c=0, and $X_n / Y_n \to X/Y$, if $c \neq 0$
22. If $\{X_n\}$ be a sequence of i.i.d. random variables with common mean μ and finite fourth moment, then $P\left\{\lim_{n\to\infty} \frac{S_n}{n} = \mu\right\} = 1.$
23. If r^{th} absolute moment of characteristic function is differentiable r times and hence show that $\Phi^r(0) = i^r \mu_r'$
24. Show that product of two characteristic function is also a characteristic function.

(8 * 2 =16 Weightage)

Part C Answer any *two* questions.

25. (a) Show that $\{X_n\}$ converges in probability to a random variable if and only if it is Cauchy in probability
(b) Show that $\{X_n\}$ Cauchy in mean implies $\{X_n\}$ Cauchy in probability.
26. State and prove Kolmogrov's SLLN's.
27. (a)State and prove Lyaponouv Central Limit Theorem
(b) Check whether $\varphi_x(t) = e^{|t|}$, $and\ \varphi x(t) = 1/2(1 + e^{it})$, are characteristic function
28. State and prove Radon- Nikodym theorem.

(2* 4 = 8 Weightage)

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