15P203 (Pages: 2) Name ....

Reg.No. ....

SECOND SEMESTER M.Sc.DEGREE EXAMINATION, JULY 2016

(CUCSS-PG)

(Mathematics)

CC 15P MT2 C08-Topology - I

(2015 Admission)

Time: 3 Hrs Maximum: 36 Weightage

Part-A

*Answer **all** questions*

*Each question has weightage **1.***

1. Find the open sets in cofinite topology space.
2. Distinguish between base and sub-base of a topological space.
3. Define embedding of a topological space into another.
4. Give an example of a connected subset C of $\mathbb{R}^2$ such that $\mathbb{R}^2 - C$ has infinitely many components.
5. Give an example of a topological space that is T_0 but not T_1.
6. Determine open sets in the set of real numbers with usual topology.
7. Prove that every compact Hausdorff space is a T_3 space.
8. Give an example of a space in which every compact subset is closed but which is not Hausdorff.
9. Define component of a space. Also find the components of a discrete space.
10. Give an example for a second countable space.
11. Find all dense subsets of $\mathbb{R}$ with usual topology.
12. Determine the topology induced by a discrete metric on a set.
13. Determine the convergent sequences in a cocountable topological space.
14. Give an example of connected space which is not locally connected.

$(14\times 1 = 14 \text{ weightage})$

Part-B

*Answer any **seven** questions*

*Each question has weightage **2.***

15. Prove that a discrete space is second countable if and only if the underlying set is countable.
16. Prove that every open cover of a second countable space has a countable subcover.
17. For a subset A of a space X, prove that $\bar{A} = A \cup A'$.
18. Prove that every separable space satisfies the countable chain condition.
19. Prove that every quotient space of a locally connected space is locally connected.
20. Prove that the product topology is the weak topology determined by the projection functions.
21. Prove that a subset of R is connected if and only if it is an interval.
22. Prove that a connected T_4 space with at least two points must be countable.

23. Prove that the composition of continuous function is continuous.
24. Suppose a topological space X has the property that for every closed subset A of X, e continuous real valued function on A has a continuous extension to X. Then show tha is normal.

(7× 2 = 14 weightage

Part-C

*Answer any **two** questions*

*Each question has weightage **4**.*

25. Prove that every regular, Lindeloffe's space is normal.
26. Prove that

    (a) Every continuous real valued function on a compact space is bounded and attains i extrema.

    (b) Every completely regular space is regular.
27. State and prove Tietze extension theorem.
28. State and prove Urysohn lemma.

(2× 4 = 8 weightag

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