16P253 (Pages: 2) Name.....................

Reg. No.....................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY-2017

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15P ST2 C06 - ESTIMATION THEORY

(Statistics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

PART A

(Answer all questions. Weightage 1 for each question)

1. Define sufficient estimator.
2. How will you quantify the information about the parameter that contained in a random variable?
3. Explain the concept of Consistency.
4. Distinguish between point estimation and interval estimation.
5. Define loss function. What are the different types of loss functions?
6. Define complete family of distributions.
7. Define MLE. Prove or disprove: MLE's are consistent.
8. Give an example of sufficient statistic which is not a minimal sufficient.
9. Distinguish between Bayesian and Fiducial intervals.
10. Define shortest length confidence interval.
11. Define one parameter exponential family of distribution. Give an example of distribution which is not a member of this family.
12. Distinguish between marginal consistency and joint consistency.

(12x1=12 weightage)

PART B

*(Answer any **eight** questions. Weightage 2 for each question)*

13. Define CAN estimator. Let $X \sim P(\lambda)$. Find CAN estimator of $e^{-\lambda}$.
14. State and prove factorization criteria on sufficiency.
15. Let $X_1, X_2, \ldots, X_n$ be i.i.d observations from $N(\theta, 1)$. Show that $T = \sum_{i=1}^{n} X_i$ where l_i' are real constants, is sufficient for θ if and only if $l_1 = l_2 =, \ldots, l_n$.
16. State and prove Cramer-Rao inequality.

17. Explain method of moment estimation. Prove or disprove: Moment estimators are always unbiased.

18. Define completeness. If T is complete, Show that any one to one function of T is complete.

19. Explain the general principle of constructing Bayesian confidence interval.

20. State and Prove Lehmann-Scheffe theorem.

21. Let $X_1, X_2, \ldots, X_n$ be a sample from $U\left(\theta - \frac{1}{2}, \theta + \frac{1}{2}\right)$. Show that

$T = \left(Min(X_1, X_2, \ldots, X_n), Max(X_1, X_2, \ldots, X_n)\right)$ is sufficient for θ but not complete.

22. Show that under squared loss function, the Bayes estimator is the mean of posterior distribution.

23. Define Pivot. Describe the method of construction of confidence interval using pivot.

24. State and prove invariance property of MLE.

(8x2=16 weightage)

PART C

*(Answer **two** questions. Weightage 4 for each question)*

25. (a). Find the 100(1-α)% shortest length confidence interval for θ when $X \sim N(\theta, 1)$.

(b). Suppose $X_1, X_2, \ldots, X_n$ be a sample from $f(x;\theta) = e^{-(x-\theta)}, x > 0, \theta > 0$ and the prior distribution of θ is $f(\theta) = e^{-\theta}, \theta > 0$. Determine the Bayes estimator of θ under squared error loss.

26. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$. Let $T_n = \bar{X}$ and $T_n^* = \frac{n}{n+1}\bar{X}$. Show that both T_n and T_n^* are CAN estimators. Which of these estimators is best estimator? Obtain the efficiency of the other estimator relative to the best estimator.

27. (a). Define UMVUE. Find UMVUE of $P(X \leq u)$, when $X \sim N(\theta, 1)$.

(b). Under regularity conditions to be stated, Prove that MLE`s are CAN estimators.

28. (a). State and prove Rao-Blackwell theorem.

(b). Find the lower bound for the variance of an unbiased estimator of θ, when $f(x;\theta) = \theta(1-\theta)^x, x = 0,1,2 \ldots$ and $0 < \theta < 1$.

(2x4=8 weightage)

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