16P202 (Pages: 3) Name..................

Reg. No.................

**SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY-2017**

(Regular/Supplementary/Improvement)

(CUCSS - PG)

**CC 15P MT2 C07 -REAL ANALYSIS-II**

(Mathematics)

(2015 Admission Onwards)

Time : 3 Hrs Maximum : 36 Weightage

## Part A

Short answer questions(1-14).

**Answer all questions. Each question has one weightage.**

1. Prove that a linear operator on $R^n$ is invertible if and only if $det[A] \neq 0$.

2. Let L(X,Y) denote the linear space of all linear maps from the vector space $X$ to $Y$. Assume $A \in L(X,Y)$ and $Ax = 0$ only when $x = 0$. Prove that $A$ is one to one. Is the converse true? Justify the claim.

3. Is it possible to find a linear operator $A$ on $R^n$ that is one to one but the range of $A$ is not all of $R^n$. Justify your answer.

4. Considering $R$ as metric space with respect to the metric $d(x,y) = |x-y|$, prove that the map $\phi : R^1 \to R^1$ defined by $\phi(x) = \frac{x-1}{2}$ has a fixed point.

5. Let f be differentiable real function in $R^n$. If $f \neq 0$, then prove that $\nabla(\frac{1}{f}) = -\frac{1}{f^2}\nabla f$.

6. Let $f = (f_1, f_2)$ be the mapping of $R^2$ into $R^2$ given by $f_1(x,y) = e^x cosy$, $f_2(x,y) = e^x siny$. Show that the jacobian of f is not zero at any point of $R^2$.

7. Find the Lebesgue outer measure of set $\{1 \pm \frac{1}{2^n}; n = 1, 2, 3...\}$.

8. Prove that outer measure is translation invariant.

9. Let $f : [1, 100] \to R$ be defined by $f(x) = e^x$. Is f measurable.

10. Prove that real valued continuous functions are Lebesgue measurable.

11. Show that if $f$ is integrable , so is $|f|$.

12. Show that monotone convergence theorem need not hold for decreasing sequence of functions.

13. Show that if $a \leq c \leq b$, then $T_a^b = T_a^c + T_c^b$.

14. Let $f$ be a non-negative measurable function. Show that $\int f = 0$ implies $f = 0$ a.e.

**Part B**

**Answer any 7 from the following questions(15-24).**

**Each question has weightage 2.**

15. Let $\Omega$ be the set of all invertible linear operator on $R^n$. Prove that $\Omega$ is an open subset of $L(R^n)$.

16. Suppose $f$ maps an open set $E \subset R^n$ into $R^m$, f is differentiable in E and there is a real number $M$ such that $\|f'(x)\| \leq M$ for every $x \in E$. Then prove that $| f(a) - f(b) | \leq | b - a |$ for all $a \in E, b \in E$.

17. Show that the operator $A^{-1}$ is linear if $A$ is an invertible linear operator.

18. State and prove linear version of implicit function theorem.

19. Prove that $m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 + mE_2$.

20. Prove that the characteristic function $\chi_E$ is measurable iff E is measurable.

21. Define Lebesgue Measure. Show that if $f$ is measurable then $f^2$ is also measurable.

22. Let $< E_n >$ be an infinite decreasing sequence of measurable sets i.e., $E_{n+1} \subset E_n$ for all n. Let $mE_1$ is finite then $m(\bigcap_{i=1}^{\infty} E_i) = lim_{n \to \infty} mE_n$.

23. If f is of bounded variation on [a,b], then prove that $f'(x)$ exists for almost all x in [a,b].

24. Let $f$ be bounded variation on [a,b]. Then prove that $T = P + N$.

## Part C

**Answer any 2 from the following 4 questions(25-28).**

**Each question has weightage 4.**

25. (a). State and prove inverse function theorem for continuously differentiable function.

    (b). Give an example of contraction mapping.

26. (a). Prove that the outer measure of an interval is its length.

    (b). Show that outer measure is sub additive.

27. (a). Construct a non-measurable set.

    (b). Show that measure satisfies monotonicity.

28. Let $f$ be an increasing real valued function on the interval [a,b]. Prove that $f$ is differentiable almost everywhere. Also prove that the derivative $f'$ is measurable and

    $\int_a^b f'(x) \leq f(b) - f(a)$.

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