16P203 (Pages: 2) Name.....................

Reg. No.....................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY-2017

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC 15P MT2 C08 - TOPOLOGY – I

(Mathematics)

(2015 Admission Onwards)

Time: 3 Hrs Maximum: 36 Weightage

Part-A

Answer all questions

Each question has weightage 1.

1. Define topological space.
2. State Urysohn's lemma.
3. Define base for a topology.
4. Give examples of two topologies on a finite set X such that one is weaker than the other.
5. Give an example of a topological space that is T_0 but not T_1.
6. Define accumulation point of a subset of a topological space. Illustrate using an example.
7. Verify whether the space of reals with usual topology is separable.
8. Find the open sets in Sierpinski space.
9. Give an example of a space in which every compact subset is closed but which is not Hausdorff.
10. Give an example for a second countable space.
11. Define quotient map from a topological space to another. Give an example.
12. Define embedding of a topological space into another.
13. Give an example for a space which is normal but not regular.
14. Verify whether the set of reals with usual topology is compact.

(14× 1 = 14 weightage)

Part-B

*Answer any **seven** questions*

Each question has weightage 2.

15. Prove that metrisability is a heriditary property.
16. Prove that semi-open interval topology is stronger than the usual topology on the set of real numbers.
17. For a subset A of a space X, prove that $\bar{A} = A \cup A'$.
18. Prove that seperable spaces satisfy countable chain condition.
19. Prove that every infinite subset of A of a compact space X has at least one accumulation point in X.
20. Prove that every second countable space is first countable.
21. Prove that a subset of R with usual topology is compact if and only if it closed and bounded.
22. Prove that product topology is the weak topology determined by the projection functions.

23. Suppose a topological space X has the property that for every closed subset A of X, every continuous real valued function on A has a continuous extension to X. Then show that X is normal.

24. Prove that open subspace of a locally connected space is locally connected.

(7× 2 = 14 weightage)

Part-C

Answer any two questions

Each question has weightage 4.

25. a) Prove that a subset of the real line is connected if and only if it is an interval.

b) Prove that every path connected space is connected.

26. a) If a space is second countable prove that every open cover of it has a countable subcover.

b) Prove that metrisability is a heriditary property.

27. Prove that

(a) Every separable space satisfies the countable chain condition.

(b) All metric spaces are T_4.

28. Prove that

(a) Every closed and bounded interval is compact.

(b) Every open surjective map is a quotient map.

(2× 4 = 8 weightage)

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