17P263 (Pages: 2) Name:………………..…….

Reg. No……………..……..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2018

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15P ST2 C08 – PROBABILITY THEORY

(Statistics)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define Axiomatic definition of probability.
2. Show that pair wise independence need not imply mutual independence.
3. Describe the properties of a distribution function.
4. Let F_n be a sequence of df's defined by $F_n(x)=0$ for $x<0$, $1-1/n$, for $0 \leq x \leq n$, 1 for $x \geq n$. Find the limiting distribution.
5. Does convergence in distribution implies convergence in probability and vice versa. Justify.
6. Define Kolmogorov's WLLN's.
7. Distinguish between convergence in probability and almost sure convergence.
8. Does convergence in rth mean implies convergence in probability. How?
9. State weak law of large numbers. Give an example.
10. What are the elementary properties of characteristic function?
11. What are super Martingale and sub Martingale?
12. Define conditional expectation.

(12 x 1 = 12 Weightage)

Part B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. Define independence of events and classes.
14. State and prove Borel 0-1 Law.
15. Let $\{X_n\}$ be sequence of random variables defined by $P(X_n=0)=1-1/n$ and $P(X_n=1)=1/n$, n=1,2,…. Show that X_n does not converge to 0 with probability 1.
16. State and prove Bernoulli's Weak Law of Large Numbers.
17. State and prove Kolmogorov's Three series theorem.
18. Prove that $\sum_{n=1}^{\infty} Var\, X_n < \infty$ implies $\sum_{n=1}^{\infty}(X_n - EX_n)$ converge almost surely

19. Briefly discuss the moment problem of characteristic functions.
20. State and prove Lindberg Levy's Central Limit Theorem.
21. Describe Doob decomposition.
22. Let $\{X_n\}$ and $\{X_n\}$ be sequence of independent random variables. If $X_n \to X$ in Law and $Y_n \to c$ in probability. Then show that $aX_n + bY_n \to aX + bc$ in Law.
23. State Martingale limit theorems.
24. Show by an example almost sure convergence implies convergence in probability.

(8 x 2 = 16 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. (a) Prove Borel Cantelli Lemma. Give application.

    (b) State and prove Kolmogorov 0-1 Law.
26. State and prove inversion theorem on characteristic functions.
27. Prove Strong Law of Large numbers.
28. State and prove Radon- Nikodym theorem.

(2 x 4 = 8 Weightage)

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