18P205 (Pages: 3) Name:……………………..

Reg. No:…………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2019

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC17P MT2 C10/CC18P MT2 C10 - ODE AND CALCULUS OF VARIATIONS

(Mathematics)

(2017 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define analyticity of a function at a point x_0
2. Locate and classify the singular points on the x-axis of the differential equation
$$(3x + 1)\, x\, y'' - (x + 1)\, y' + 2y = 0$$
3. Find the indicial equation and its roots for the differential equation
$$x^3 y'' + (\cos 2x - 1) y' + 2xy = 0$$
4. Evaluate $\lim_{a\to\infty} F\left(a, a, \frac{1}{2}, \frac{-x^2}{4a^2}\right)$
5. Determine the nature of the point x = ∞ for the Bessel's equation
$$x^2 y'' + x y' + (x^2 - p^2)\, y = 0$$
6. State the orthogonality property of Legendre polynomials.
7. Replace the differential equation $y'' - x^2 y' - xy = 0$ by an equivalent system of first order equations.
8. Define the Wronskian of two solutions of the homogenous system
$$\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$$
9. Define autonomous system and give an example.
10. Describe the phase portrait of the system $\begin{cases} \frac{dx}{dt} = -x \\ \frac{dy}{dt} = -y \end{cases}$
11. Determine the auxiliary equation of the autonomous system $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$
12. State Sturm comparison theorem.
13. Define critical point and write the list of different types of critical points.
14. Find the extremal for the intergral $\int_{x_1}^{x_2} \frac{\sqrt{1+(y')^2}}{y}$

(14 x 1 = 14 Weightage)

Turn Over

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Consider the differential equation $y' = 2xy$, find a power series solution of the form $y = \sum a_n x^n$, recognise the resulting series as the expansion of a familiar function and verify it by solving the equation directly.
16. Find a Frobenius series solution of the differential equation $4x^2y'' - 8x^2y' + (4x^2 + 1)y = 0$.
17. Consider the differential equation $x(1 - x)y'' + [p - (p + 2)x]y' - py = 0$, where p is a constant. If p is not an integer, find the general solution near x = 0 in terms of hypergeometric functions.
18. Let $J_p(x)$ be the Bessel function of the first kind of order p. Prove the following
    (a) $2J_p'(x) = J_{p-1}(x) - J_{p+1}(x)$
    (b) $2p\, J_p(x) = x[J_{p-1}(x) + J_{p+1}(x)$
19. Let $P_n(x)$ be the n^{th} Legendre polynomial, Prove that
    $(n + 1) P_{n+1}(x) = (2n + 1)\, x\, P_n(x) - n\, P_{n-1}(x)$
20. Find the first three terms of the Legendre series of $f(x) = e^x$
21. For the linear system $\begin{cases} \frac{dx}{dt} = \ \ x \\ \frac{dy}{dt} = -y \end{cases}$

    Find :

    (a) The differential equation of the paths.

    (b) Solve the equation found in (a) and sketch a few of the paths showing the direction of increasing it.

    (c) Discuss the stability of the critical point.
22. Define simple critical points of non linear systems.

    Show that $(0, 0)$ is simple critical point of $\begin{cases} \frac{dx}{dt} = -2x + 3y + xy \\ \frac{dy}{dt} = -x + y - 2xy^2 \end{cases}$
23. Show that the function $f(x, y) = xy$
    (a) Satisfies a Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le d$
    (b) Does not satisfy Lipschitz condition on the entire plane.
24. Find the point on the plane ax + by + cz = d that is nearest the origin by the method of Lagrange multipliers.

(7 x 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Find two independent Frobenius series solutions of the differential equation
    $xy'' + 2y' + xy = 0$
26. Derive the Rodrigue's formula for Legendre polynomials, $P_n(x)$
27. Find the general solution of the linear system $\begin{cases} \frac{dx}{dt} = -3x + 4y \\ \frac{dy}{dt} = -2x + 3y \end{cases}$
28. State and prove Picard's theorem regarding the existence and uniqueness of solutions of an initial value problem.

(2 x 4 = 8 Weightage)

****