D 71329 (Pages : 2) Name..

Reg. No..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Mathematics

MT 3C 13—TOPOLOGY II

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*

Each question has weightage 1.

1. Prove that if a product is non-empty, then each projection function is onto.
2. Let C_i be a closed subset of a space Xi, for $i \varepsilon$ I. Prove that $\prod_{i \in I} C_i$ is a closed subset of $\prod_{i \in I} X_i$ with respect to the product topology.
3. Define a cube and a Hilbert cube.
4. Give an example of a topological property which is not productive.
5. Prove that if the evaluation map of the family of functions is one-to-one, then that family distinguishes points.
6. Give an example of a matric space which is not second countable.
7. Let f and f^1 be two paths in a space X such that f is path homotopic to f^1. Prove that f^1 is path homotopic to f.
8. If X is any convex subset of R^n, prove that Π_1 (X, x_0) is the trivial group.
9. Prove that the map $P : R \to S^1$ given by $P(x) = (\cos 2\pi n, \sin 2\pi n)$ is a covering map.
10. Prove that a continuous function from a compact metric space into another metric space is uniformly continuous.
11. If a space X is regular and locally compact at a point $x \in X$, then prove that x has a local base consisting of compact neighbourhoods.
12. Describe the one-point compactification of a topological space X.
13. Give an example of a metric which is bounded but not totally bounded.
14. Define nowhere dense set in a topological space X. Give an example of a nowhere dense set in the real line with the usual topology.

(14 × 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *quesions.*
Each questions has weightage 2.

15. Let A be a closed subset for a normal space X and suppose $f : A \to (-1, 1)$ is continuous. Prov that there exists a continuous function $F : X \to (-1, 1)$ such that $F(x) = f(x)$ for all $x \in A$.
16. If the product is non-empty, then prove that each co-ordinate space is embeddable in it.
17. Prove that a product of topological spaces is regular if each co-ordinate sapce is regular.
18. State and prove the embedding lemma.
19. Let X be path connected and x_0 and x_1 be two points of X. Prove that π_1^{*} (X, x_0) is isomorphic t π_1 (X, x_1).
20. Let A be a strong deformation retract of a space X. Let $a_0 \in A$ Prove that the inclusion map

    $j : (A, a_0) \to (X, a_0)$

    induces an isomorphism of fundamental groups.
21. Let $\{X_i : i \in I\}$ be an indexed family of non-empty compact spaces and let x be their topologic product. Prove that X is compact.
22. Let X be a Hausdorff space and let Y be a dense subset of X. If Y is locally compact in the relati topology on it, prove that Y is open in X.
23. Prove that a metric space is compact if and only if it is complete and totally bounded.
24. Prove that equivalence of cauchy sequences is an equivalence relation on the set of all cauch sequences in a metric space (x, d).

(7 × 2 = 14 weightag

Part C

Answer any **two** *questions.*
Each question has weightage 4.

25. Prove that metrisability is a countably productive property.
26. State and prove Urysohn's metrisation theorem.
27. Let $P : E \to B$ be a covering map, let $P(e_0) = b_0$. Prove that any path $f : [0, 1] \to B$ beginning at has a unique lefting to a path $\tilde{f}$ in E beginning at e_0.
28. Prove that the one-point compactification of a space is Hausdorff if and only if the space is local compact and Hausdorff.

(4 × 2.= 8 weightag