12

71327 (Pages : 3) Name..................................

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Mathematics

MT 3C 11—COMPLEX ANALYSIS

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Show that if a linear transformation has ∞ for its only fixed point, then it is a translation.
2. State the symmetry principle.
3. Compute the cross ratio :

   $(i, 0, -1, \infty)$.

4. Find the image of the hyperbola $\left\{z = x + iy : x^2 - y^2 = 1\right\}$ under the map $f(z) = z^2$.
5. State the Cauchy's Integral Formula.
6. Compute $\int_r \frac{\cos z}{z} dz$, where $r(t) = e^{it}, 0 \le t \le 2\pi$.
7. What is the nature of the singularity of e^z at $z = \infty$?
8. State general form of Cauchy's theorem.
9. Find the nature of the singularity of the function $\frac{1}{\sin^2 z}$ at $z = 0$.
10. Let u be a real valued piecewise continuous function on $[0, 2\pi]$. Define the Poisson integral of u.
11. Find the roots of the equation :

    $z^4 - 6z + 3 = 0$

    in the annulus $\left\{z : 1 < |z| < 2\right\}$.

Turn over

12. Obtain the power series expansion of $\frac{1}{z}$ about $z = 1$ in the disk $\{z : |z-1| < 1\}$.

13. Prove that the sum of the residues of an elliptic function is zero.

14. Show that there does not exist an elliptic function with a single simple pole.

(14 × 1 = 14 weighta

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Prove that a linear transformation carries circles into circle.

16. Describe the mapping properties of $W = e^z$.

17. State and prove Schwarz's lemma.

18. Let r be a closed rectifiable curve. For any point 'a' not on r define $n(r, a)$. Show that $n(r, a$ always an integer.

19. Show that if $f(z)$ is analytic in a region Ω and statistics the inequality $|f(z)-1| < 1$ on Ω, th

$$\int_r \frac{f'(z)}{f(z)}\, dz = 0, \text{ for every closed curve } r \text{ in } \Omega.$$

20. Obtain the Laurent series expansion of $\frac{1}{z(z-1)}$ in :

(i) $0 < |z| < 1$. (ii) $|z| > 1$.

21. State and prove the Residue theorem.

22. If $f(z)$ is analytic in a region Ω and has no zeros in Ω, prove that $\log|f(z)|$ is harmonic in

23. Suppose f has an isolated singularity at $z = a$. If $\lim_{z\to a}(z-a)f(z) = 0$, show that $z = a$ is a remov singularity.

24. Show that any even elliptic functions with periods w_1 and w_2 can be expressed in the form :

$$C \prod_{k=1}^{n} \frac{p(z)-p(a_k)}{p(z)-p(b_k)}$$

where C is a constant.

(7 × 2 = 14 weight

Part C

Answer any **two** *questions.*
Each question carries 4 weightage.

25. State and prove Cauchy's theorem for a rectangle.

26. Using Residue theorem, evaluate the integral $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}$, where $a > 1$.

27. Derive the formula for the Weierstrass elliptic function in the form :

$$p(z) = \frac{1}{z^2} + \sum_{w \neq 0} \left(\frac{1}{(z-w)^2} - \frac{1}{w^2} \right).$$

28. Derive the Poisson integral formula for harmonic functions.

(2 × 4 = 8 weightage)