15P301 (Pages : 2) Name......................

Reg. No....................

THIRD SEMESTER M.Sc DEGREE EXAMINATION NOVEMBER 2016-17

(CUCSS)

(Mathematics)

CC15PMT3C11- COMPLEX ANALYSIS

(2015 Admission)

Time: Three Hours Max : 36 weightage

PART A

Answer **ALL** questions

Each question carries 1 weightage

1. Show that linear transformation preserves cross ratio.
2. If $T_1(z) = \frac{z+2}{z+3}$ and $T_2(z) = \frac{z}{z+1}$, find $T_1T_2(z)$, $T_2T_1(z)$.
3. Find the cross ratio of $(i, 0, -1, \infty)$.
4. Compute $\int_\gamma x\, dz$ where γ is a directed line segment from 0 to 1+i.
5. State cauchy's theorem in a disc.
6. Prove that $n(\gamma, a)$ is a constant in each of the regions determined by γ.
7. State Weierstrass theorem on essential singularity .
8. Evaluate $\int_{|z|=1} \frac{e^z}{z} dz$.
9. Find the residues of the function $f(z) = \frac{e^z}{(z-a)4}$ at $z = a$.
10. Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lies between the circles $|z| = 1$ and $|z| = 2$.
11. Define simply connected region. Give an example of a simply connected region.
12. Expand $\frac{1}{(z-1)(z-2)}$ as a laurentz series in the region $1 < |z| < 2$.
13. Prove that sum of the elliptic function at its poles is zero.
14. Find the harmonic conjugate of the function $e^x\, siny$. **(14x1=14 weightage)**

PART B

Answer any **SEVEN** questions

Each question carries 2 weightage

15. Describe the mapping properties of $w = e^z$.
16. Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

17. If f is a continuous complex valued function defined on $[a, b]$, then prove that $\left|\int_a^b f(t)\, dt\right| \leq \int_a^b |f(t)|\, dt.$

18. Prove that a bounded entire function is constant.

19. If $f(z)$ is analytic in Ω, then prove that $\int_\gamma f(z)\, dz = 0$ for every cycle γ which is homologues to zero in Ω

20. State and Prove Rouche's theorem.

21. State and Prove Hurwitz's theorem.

22. Evaluate $\int_0^\infty \frac{dx}{1+x^2}$.

23. Derive the Legendre relation $n_1\omega_2 - n_2\omega_1 = 2\pi i$.

24. Show that any even elliptic function with periods $\omega_1 and\ \omega_2$ can be expressed in the form $C \prod_{k=1}^{n} \frac{P(z)- P(a_k)}{P(z)- P(b_k)}$ where C is a constant.

(7×2=14 weightage)

PART C

Answer any **TWO** questions

Each carries 4 weightage

25. State and prove Cauchy's theorem for a rectangle.

26. Derive Poisson's integral formula for harmonic function.

27. State and prove mean value property of harmonic functions.

28. Explain the construction of the modular function $\lambda(\tau)$ and show that λ is invariant under the congruence sub group modulo 2.

(2×4=8 weightage)

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