16P359 (Pages: 2) Name……………………..

Reg. No………………..

TTHIRD SEMESTER M.Sc. DEGREE EXAMINATION, OCTOBER 2017

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15P ST3 C12 - TESTING OF STATISTICAL HYPOTHESES

(Statistics)

(2015 Admission Onwar)

Time: Three Hours Maximum: 36 Weightage

Part A

I. Answer all questions.

1. Distinguish between Parametric and Nonparametric tests.
2. State generalized Neyman-Pearson Lemma.
3. Define (a) p value (b) Level of Significance.
4. Define (a) UMP test (b) UMP Unbiased test.
5. Explain locally most powerful tests.
6. Define empirical distribution and discuss its properties.
7. Discuss the construction of α-similar tests with Neyman-structure.
8. Explain Sequential estimation.
9. Discuss the merits of Wilcoxon signed rank test over sign test.
10. How do you determine the stopping bounds of an SPRT?
11. Derive the asymptotic distribution of likelihood ratio statistic.
12. Define OC function of SPRT. Point out its uses. (12 X 1=12 Weightage)

Part B

II. Answer any eight questions. .

13. Let $X \sim N(\mu, \sigma^2)$ Show that there does not exist UMP test of $H_0: \sigma = {\sigma_0}^2 \ vs \ H_1: \sigma \neq {\sigma_0}^2$.

14. If $\ell(x)$ is the likelihood ratio for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_1$ where θ is a scalar, then show that the asymptotic distribution of $-2 \log \ell(x)$ is $\chi^2(1)$.

15. To test $H_0: \mu = 10 \ vs \ H_1: \mu = 15$ in $N(\mu, \sigma^2)$. Find the minimum sample size to ensure $\alpha = 0.05$, $\beta = 0.025$ if it is given that $\sigma = 5$

16. Prove that SPRT terminates with probability one.

17. Explain Mann-Whitney test for $H_0: F(x) = G(x) \ vs \ H_1: F(x) < G(x)$.

18. Show that if most powerful test exists, it is a function of sufficient statistic.

19. Explain how you can use ordinary sign test in the case of paired samples.

20. State and prove Wald's identity

21. Explain median test. What is the null distribution of test statistic?

22. Give an example for a distribution with MLR property and one without it .Justify.

23. Define ASN. Obtain its expression.

24. Let X_i, $(i = 1,2, \ldots . n.)$ be a random sample of size n taken from a population with density Function $f(x,\theta) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2}\ x \in R$ .Obtain a LMP test for $H_0\colon \theta \leq 0\ vs H_1\colon \theta > 0$

(8 X 2=16 Weightage)

Part C

III. Answer any two questions.

25. (a) An urn contains 10 marbles of which *M* are white and *10-M* are black. To test $H_0\colon M = 5\ vs\ H_1\colon M = 6$, One drawn three marbles from the urn without replacement. The null hypothesis is rejected if the sample contains 2 or 3 white marbles . Find the size and power of the test.
(b) Show that the most powerful test of the Neyman-Pearson lemma for simple hypothesis against simple alternative is strictly unbiased, if $0 < \alpha < 1$.

26. Let $X \sim N(\mu_1, \sigma^2{}_1)$ and $Y \sim N(\mu_2, \sigma^2{}_2)$. Assume that X and Y are independent. Obtain UMP unbiased test for $H_0\colon {\sigma_1}^2 = {\sigma_2}^2\ vs\ H_1\colon {\sigma_1}^2 \neq {\sigma_2}^2$ based on two independent sets of samples. Is this test coincides with likelihood ratio test? Justify.

27. Derive the SPRT for testing $H_0\colon \mu = \mu_0\ vs\ H_1\colon \mu = \mu_1$ for a normal population $N(\mu, 1)$ with strength ***(α,β)***. Derive the Expressions for OC and ASN function in this case.

28. Compare Chi-square test and Kolmogorov-Smirnov test for goodness of fit by clearly explaining the two tests.

(2 X 4 = 8 Weightage)

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