THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019
(Regular/Supplementary/Improvement)
(CUCSS-PG)
(Statistics)
CC15P ST3 C11 - STOCHASTIC PROCESSES
(2015 Admission onwards)

Time : Three Hours   Maximum : 36 Weightage

Part A
Answer *all* questions. Each question carries 1 weightage.

1. Define stationary process.

2. Define continuous time continuous state stochastic process. Give examples.

3. Explain one step TPM of a Markov chain.

4. What do you mean by equivalence class? Give an example.

5. State and prove the memory less property of exponential distribution.

6. Write down the postulates of a Counting process.

7. Write the Chapman-Kolmogorov's equations of a continuous time Markov process.

8. Define Birth-Death process.

9. Define renewal process.

10. Show that the renewal function $m(t) = \sum_{n=1}^{\infty} F_n(t), \forall t$, where $F_n(t) = P(S_n \leq t), n \geq 1, \forall t$

11. Define queueing process.

12. Define Brownian motion process.

(12 x 1 = 12 Weightage)

Part B
Answer any *eight* questions. Each question carries 2 weightage.

13. Show that periodicity of a Markov chain is a class property.

14. Describe a Poisson Process and derive its distribution.

15. Show that state i is recurrent if $\sum_{n=1}^{\infty} p_{ii}^{(n)} = \infty$ and is transient if $\sum_{n=1}^{\infty} p_{ii}^{(n)} < \infty$

16. Show that in an irreducible MC, all the states are of the same type.

17. Explain Yule process.

18. Derive the Chapman – Kolmogorov equation for continuous time Markov chain.

19. If $\{N(t)\}$ is a Poisson process, derive the auto-correlation between N(t) and N($t+s$), $t,s > 0$

20. Let S_n be the waiting time for the occurrence of nth renewal and m(t) be the renewal function of renewal process. Show that $E\{S_{N(t)+1}\} = E(X_1) \{1 + m(t)\}$

21. Explain the semi-Markov process.

22. Obtain the steady state probabilities of M/M/1 model.

23. State and prove elementary renewal theorem.

24. Explain the first hitting time distribution of Brownian motion process.

(8 x 2 = 16 Weightage)

Part C

Answer any *two* questions. Each question carries 4 weightage.

25. Describe Branching process. Find the mean and variance of the G.W. branching process.

26. (a) Describe various class properties.

(b) Explain the class property of Markov chain with transition probability matrix .

$$\begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

27. Obtain the steady state probability distribution of M/G/1 model.

28. Explain:

(a) Non-homogeneous Poisson process.

(b) Compound Poisson process.

(c) Conditional mixed Poisson process.

(2 x 4 = 8 Weightage)

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