

C 5129

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Name.....

Reg. No.....

FOURTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT)
EXAMINATION, MAY 2016

(UG-CCSS)

Core Course—Mathematics

MM 4B 04—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* questions.

1 Find the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

2 Evaluate $\int_0^{\pi} \frac{\sin t}{2 - \cos t} dt$.

3 Evaluate $\frac{d}{dt} (\tanh \sqrt{1+t^2})$.

4 Write the parametric equations of the circle $x^2 + y^2 = 1$.

5 Find the vertices of the hyperbola $\frac{y^2}{4} - \frac{x^2}{5} = 1$.

6 Examine whether $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$ represents a parabola, ellipse or hyperbola.

7 Define absolute convergence.

8 Show that x^2 grows faster than $\ln x$ as $x \rightarrow \infty$.

9 Find y if $\ln y = 3t + 5$.

10 Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$.

11 Find the Taylor polynomial of order 1 generated by $f(x) \ln x$ at $a = 1$.

12 Examine whether $\sum_{n=1}^{\infty} n^2$ converges or diverges.

(12 × ¼ = 3 weightage)

Turn over

II. Answer *all* questions.

13 Examine whether the series

$$5 + \frac{2}{3} + 1 + \frac{1}{7} + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \dots + \frac{1}{k!} + \dots \text{converges.}$$

14 Prove that the alternating series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ converges.

15 Find $\frac{dy}{dx}$ if $y = x^x$, $x > 0$.

16 Evaluate $\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \frac{4 \cos \theta}{3 + 2 \sin \theta} d\theta$.

17 Graph the set of points whose polar co-ordinates satisfy the conditions $r \leq 0$ and $r \leq 0$, $\theta = \frac{\pi}{4}$.

18 Evaluate $\frac{d}{dx} \ln_{10} (3x + 1)$.

19 Evaluate $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$.

20 Find the Maclausin's series for $f(x) = \frac{1}{1+x}$.

21 For what values of x do the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ converges.

(9 × 1 = 9 weightage)

III. Answer any *five* questions.

22 Find the Maclaurin's series for $f(x) = \sin 3x$.

23 Find the centroid of the first quadrant of the astroid $x = \cos^3 t$, $y = \sin^3 t$, $0 \leq t \leq 2\pi$.

24 Evaluate $\int_0^1 \sin h^2 x dx$.

25 Find the Taylor polynomial generated by $f(x) = \cos x$ at $x = 0$.

26 Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

27 Graph the curve $r^2 = 4 \cos \theta$.

- 28 The co-ordinate axes are to be rotated through an angle α to produce an equation for the curve $2x^2 + \sqrt{3}xy + y^2 - 10 = 0$ and has no cross production. Find α and the new equation. Identify the curve.

(5 × 2 = 10 weightage)

IV. Answer any *two* questions.

- 29 Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

30 Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos \theta)$.

31 Find the length of the cardioid $r = 1 - \cos \theta$.

(2 × 4 = 8 weightage)