16P404 (Pages:3) Name…….

Reg. No……..

FOURTH SEMESTER M. Sc DEGREE EXAMINATION, MARCH 2018

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT4E05: OPERATIONS RESEARCH

(Mathematics)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** the questions. Each question carries ***1*** weightage.

1. Define the term chain in a graph and when it does become a cycle?
2. Show that if $\{x_i\}$ and $\{y_i\}$ are two flows in a graph, then $\{ax_i + by_i\}$ where a and b are real constants, is also a flow.
3. Write the dual of Minimize $(4 + 2\lambda)x_1 + 3(1 + \lambda)x_2$

   Subject to

   $$3x_1 + 4x_2 \leq 12,$$
   $$3x_1 + 3x_2 \leq 10$$
   $$4x_1 + 2x_2 \leq 8,$$
   $$x_1, x_2 \geq 0,\ \lambda \geq 0$$
4. Explain the term parametric linear programming problem.
5. Show that the following two problems of quadratic programming bear the primal and dual relationship with each other.

   Primal: minimize $\mathrm{PX} + X'CX$ subject to $\mathrm{AX} \geq \mathrm{B}, \mathrm{X} \geq 0$

   Dual: maximize $\mathrm{B'Y} - \mathrm{X'CX}$ subject to $\mathrm{P'} + 2\mathrm{CX} - \mathrm{A'Y} \leq 0, \mathrm{Y} \geq 0$
6. Write a short note on sensitivity analysis.
7. Analyze the optimality conditions of quadratic programming problem.
8. Show that in the problem

   Maximize $3x_1 + 6x_2 - 4x_1x_2 - 3x_1^2 - 2x_2^2$

   Subject to

   $$3x_1 + 2x_2 \leq 4,$$
   $$x_1 + x_2 \leq 1,$$
   $$x_1, x_2 \geq 0$$

   The objective function is strictly concave.
9. Use geometric programming to find the dimensions of the rectangle of maximum area inscribed in a circle of radius r
10. Explain Fibonacci search plan

11. State the necessary conditions for a serial multistage problem to be decomposable.

12. Define unimodal function.

13. Explain method of false position.

14. Find the maximum of $f(x) = -0.55 + 3x - x^2$ by Rosenbrock algorithm starting x = 0, h = 1

(14 x 1 = 14 weightage)

Part B

Answer ***all*** the questions. Each question carries ***2*** weightage.

15. Find the minimum spanning tree in the following undirected graph. Arc (i, j) denoted as (i,j)

Arc	(1,2)	(1,3)	(1,4)	(2,3)	(2,8)	(2,10)	(3,4)	(3,8)	(4,5)	(4,6)
Length	7	4	8	3	9	14	4	1	15	12

Arc	(4,8)	(5,6)	(5,7)	(6,7)	(6,8)	(6,9)	(7,9)	(8,9)	(8,10)	(9,10)
Length	10	4	1	2	20	16	18	3	4	12

16. Show that the optimal solution of the following problem for $\lambda = 0$ remains optimal for $0 \le \lambda \le \frac{2}{3}$

$$\begin{aligned} \text{Maximize} \quad & 3x_1 + 6x_2 \\ \text{Subject to} \quad & (1+2\lambda)x_1 \le 4, \\ & 3(1-\lambda)x_1 + 2x_2 \le 18, \\ & x_1, x_2 \ge 0 \end{aligned}$$

17. Solve the following problem using simplex method, and analyze the effects of the changes using sensitivity analysis method.

$$\begin{aligned} \text{Maximize} \quad & 4x_1 + 8x_2 \\ \text{Subject to} \quad & x_1 + 2x_2 \ge 20, \\ & 2x_1 + 2x_2 \le 100 \\ & x_1 - 3x_2 \le 0, \end{aligned}$$

18. Solve by the method of quadratic programming.

$$\begin{aligned} \text{Maximize} \quad & 6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2 \\ \text{subject to} \quad & x_1 + x_2 \le 2 \\ & x_1, x_2 \ge 0 \end{aligned}$$

19. Describe the effect of deletion of the variables in the optimal solution of an LP problem.

20. Explain the dynamic programming problem with single additive constraint and additively separable return.

21. Briefly describe the computational algorithm of golden section search plan.

22. Maximize $\sum_{n=1}^{4}(4u_n - nu_n^2)$

Subject $\sum_{n=1}^{4} u_n = 10, u_n \ge 0$

23. Describe method of axial directions in multidimensional search.

24. Prove that

"If the nonzero vectors $s_1, s_{2,\dots} s_n$ are conjugate with respect to the symmetric and positive definite matrix H, then the vectors are linearly independent"

(7 x 2 = 14 weightage)

Part C

Answer any ***two*** questions. Each question carries ***4*** weightage.

25. Solve the parametric problem $f(\lambda) = (1+\lambda)x_1 + (-2-2\lambda)x_2 + (1+5\lambda)x_3$

$$\begin{aligned} \text{Subject to} \quad & 2x_1 - x_2 + 2x_3 \le 2 \\ & x_1 - x_2 \le 3 \\ & x_1 + 2x_2 - 2x_3 \le 4 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

26. Maximize $8x_1 - 10x_2 - x_1^2 - x_2^2$

Subject to

$3x_1 - 2x_2 \le 6,$

$x_1, x_2 \ge 0$ by the method of quadratic programming and verify graphically.

27. How does K-T theory leads to the primal dual concept in the optimization theory? Explain.

28. Explain the method of steepest descent. Starting from the point (0, 0) carry out at the most five iterations for the following problem using the method steepest descent.

Maximize $f(x) = -3(x_1 - 2)^2 - 4(x_2 - 3)^2 - 2(x_3 + 5)^2$

(2 x 4 = 8 weightage)

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