D 72889 (Pages : 3) Name..... 49

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Mathematics

MAT 1C 05—DISCRETE MATHEMATICS

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Questions) (1 – 14)

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Define strict partial order and give an example of it. If R is a partial order on a set X, then prove that $R - \{(x, x) : x \in X\}$ is a strict partial order on X.
2. Prove that intersection of two chains is a chain.
3. Let $(X, +, ., ')$ be a Boolean algebra. Prove that $x + x = x$ for all $x \in X$.
4. Prepare the table of values of the following function :

   $f(x_1, x_2, x_3) = x_1' \, x_2 \, (x_1' + x_2 + x_1 x_3)$.
5. Define Chromatic number of a graph. Find the chromatic number of P_5.
6. Prove that every graph with n vertices and k edges has at least $n - k$ components.
7. If every vertex of a graph G has degree at least 2, then prove that G contains cycle.
8. Prove that every tree with at least two vertices has at least two end leaves.
9. Define Connectivity of a graph. Prove that $k(K_n) = n - 1$.
10. Is every subgraph of a non-planar graph non-planar ? Justify your answer.
11. Let u be a string on the alphabet Σ. Prove that $|u^n| = n|u|$ for all $n = 1, 2, ...$
12. Let $G = (\{S\}, \{a, b\}, S, P)$ be a grammar with productions P given by

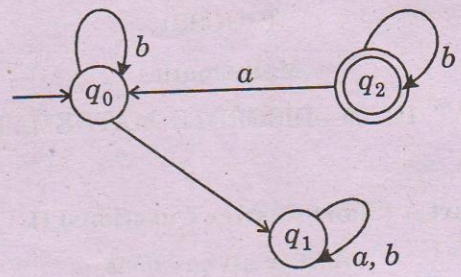
    $S \to aA, A \to bS, S \to \lambda$.

    Give a simple description of the language generated by G.

Turn over

13. Define non-deterministic acceptor and give an example of it.

14. Find the set of strings accepted by the following deterministic finite acceptor.



(14 × 1 = 14 weighta

Part B

Answer any **seven** *from the following ten questions* (15 – 24).
Each question carries weightage 2.

15. Let $(X, +, ., ')$ be a Boolean algebra. Prove that the corresponding lattice $(X, \leq)$ is complemer and distributive.

16. Let $(X, +, ., ')$ be a finite Boolean algebra. Prove that every non-zero element of X contains at l one atom.

17. Prove that the characteristic numbers of a symmetric Boolean function completely determine

18. Prove that Petersen graph has diameter 2.

19. Prove that every, u, v-walk contains a u, v-path.

20. Let G be a graph. Prove that

$$\delta(G) \leq \frac{2e(G)}{n(G)} \leq \Delta(G),$$

here $e(G)$ and $n(G)$ denote the number of edges and vertices in G respectively.

21. Draw a graph G with $k(G) < k'(G) < \delta(G)$.

22. Is Euler's formula valid for a disconnected graph ? Justify your answer.

23. Find a grammar that generate the language $\{a^{n+2} b : n \geq 1\}$.

24. Construct a nondeterministic acceptor that accepts the language $\{ab, abc\}^*$.

(7 × 2 = 14 weig

Part C

Answer any **two** *from the following four questions.* (25 – 28)
Each question carries weightage 4.

25. (a) Let $(X, +, ., ')$ be a finite Boolean algebra. Prove that every element of X can be uniquely expressed as sum of atoms.

(b) Write the Boolean function :

$$f(a, b, c) = a + b + c'.$$

in their disjunctive normal form.

26. (a) Prove that a graph is a bipartite graph if and only if it has no odd cycle.

(b) Let G be a graph. Prove that

$$\sum_{v \in V(G)} d(v) = 2e(G).$$

27. Let G be an n-vertex graph with $n \geq 1$. Prove that the following are equivalent :

(a) G is connected and has no loops.

(b) G is connected and has $n - 1$ edges.

(c) G has $n - 1$ edges and no cycles.

(d) G has no loops and has, for each $u, v \in V(G)$, exactly one u, v-path.

28. Define equivalent grammars. Prove that the grammar $G = (\{S\}, \{a, b\}, S, P)$ with productions P given by :

$$S \to SS \mid SSS \mid aSb \mid bSa \mid \lambda,$$

is equivalet to the grammar $G' = (\{S\}, \{a, b\}, S, P')$ with production P' given by :

$$S \to SS \mid aSb \mid bSa \mid \lambda.$$

(2 × 4 = 8 weightage)