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D 72885 (Pages : 3) Name.....................................

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Mathematics

MT 1C 01—ALGEBRA – I

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*

Each question carries 1 *weightage.*

1. Define the group of symmetries of a subset S of R^2 in R^2 and give an example of it.
2. Find the subgroups generated by $\{4,6\}$ in Z_{12}.
3. Find all subgroups of $Z_2 \times Z_2 \times Z_4$ that are isomorphic to the Klein 4-group.
4. Let $u = 1101010111$ and $v = 0111001110$. Find $u + v$ and $wt\ (u - v)$.
5. Find all abelian groups, upto isomorphism of order 16.
6. Let X be a G-set for $x_1, x_2 \varepsilon$ X, let $x_1 \sim x_2$ iff there exists g ε G such that $gx_1 = x_2$. Show that $\sim$ is a symmetric relation on X.
7. Find all Sylow 3-subgroups of S_4.
8. Show that the center of a group of order 8 is non-trivial.
9. Find the reduced form and the inverse of the reduced form of the word $a^2a^{-3}b^3a^4c^4c^2a^{-1}$.
10. Define the evaluation homomorphism.
11. Find all generators of the cyclic multiplicative group of units of the field Z_7.
12. Let Q be the skew field of quaternions. Write the element $(i+j)^{-1}$ in the form $a_1 + a_2 i + a_3 j + a_4 k$ for $a_i\ \varepsilon\ R$.
13. Find all zeros of $x^3 + 2x + 2$ in Z_7.
14. Find all ideals N of Z_{12}.

(14 × 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *questions.*
Each question carries 2 weightage.

15. Find the order of the element $(2,0)+\langle(4,4)\rangle$ in $Z_6 \times Z_8 / \langle(4,4)\rangle$.

16. Show that a subgroup M of a group G is a maximal normal subgroup of G iff G/M is simple.

17. Give isomorphic refinements of the two series :

$\{0\} < 60Z < 20Z < Z$ and $\{0\} < 245Z < 49Z < Z$.

18. Show that if $H_0 = \{e\} < H_1 < H_2 < \ldots < H_n = G$ is a subnormal series for a group G, and if $\frac{H_i}{H}$

is of finite order $S_i + 1$. Then G is of finite order $S_1, S_2, \ldots S_n$.

19. Let G be a finite group and X a finite G-set. Show that if r is the number of orbits in X unde then :

$$r \cdot |G| = \sum_{g \varepsilon G} |X_g|.$$

20. Show that for a prime number p, every group G of order p^2 is abelian.

21. Show that there are no simple groups of order 255.

22. Show that $\left(a,b : a^3 = 1, b^2 = 1, ba = a^2 b\right)$ gives a non-abelian group of order 6.

23. Demonstrate that $x^4 - 22x^2 + 1$ is irreducible over Q.

24. Give the addition and multiplicative tables for the group algebra $Z_2(G)$, where $G = \{a, b\}$ is cyc order 2.

(7 × 2 = 14 weigh

Part C

Answer any **two** *questions.*
Each question carries 4 weightage.

25. Show that the group $Z_m \times Z_n$ is isomorphic to Z_{mn} iff m and n relative prime. Deduce that i any integer written as $n = \left(p_1^{r_1}\right)(p_2)^{r_2} \ldots (p_m)^{r_m}$, where p_i's are distinct primes then Z_n is isomo to $Z_{(p_1)^{r_1}} \times Z_{(p_2)^{r_2}} \times \ldots \times Z_{(p_m)^{r_m}}$.

26. Let H be a subgroup of a group G. Prove that the following conditions are equivalent :

(i) $ghg^{-1} \varepsilon H$ for all $g \varepsilon G$ and $h \varepsilon H$.

(ii) $gHg^{-1} = H$ for all $g \varepsilon G$.

(iii) $gH = H_g$ for all $g \varepsilon G$.

Give an example of a subgroup H of a group G which does not satisfy condition (iii).

27. State and prove Cauchy's theorem using Cauchy's theorem, prove that a finite group G is a p-group iff $|G|$ is a power of p.

28. State and prove Eisenstein's theorem using Eisenstein's theorem, prove that the cyclotomic polynomial :

$$\phi_p(x) = \frac{x^p - 1}{x - 1} = x^{p-1} + x^{p-2} + \ldots + x + 1$$

is irreducible over Q for many prime p.

(2 × 4 = 8 weightage)