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FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Mathematics

MT 1C 02—LINEAR ALGEBRA

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Type)

Answer **all** *questions.*

Each question has weightage 1.

1. Let V be a vector space over a field F and $1 \in F$. Prove that $(-1) \cdot v = -v$ for all $v \in V$.
2. Show that $U = \{(x,0) : x \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.
3. Verify whether $\{(1,2,3), (1,3,1)\}$ is a basis for $\mathbb{R}^3$.
4. Give an example of a 2-dimensional subspace of $\mathbb{R}^3$.
5. Find the co-ordinate vector of $(1,2,3) \in \mathbb{R}^3$ with respect to the basis $\{(1,1,0), (1,0,1), (0,1,1)\}$.
6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x,y) = (x+1, y+1)$. Verify whether T is a linear transformation.
7. Let $W = \text{span}\{(1,0,0),(1,1,0)\}$. Find a non-zero linear function in W^0.
8. Find the characteristic polynomial of $\begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix}$.
9. Find the characteristic values of $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.
10. Verify whether $W = \{(x,0,0) : x \in \mathbb{R}\}$ is an invariant subspace of $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by : $T(x,y,z) = (x+y, y+z, z)$.

Turn over

11. Let $W_1 = \text{span}\{1,2,1\}$ and $W_2 = \text{span}\{(2,1,1),(1,-1,0)\}$. Verify whether $W_1 + W_2$ is a direct sum.

12. Verify whether $T:\mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x,y) = (x+y,0)$ is a projection.

13. Let V be an inner product space. Prove that $\| c\alpha \| = | c | \cdot \| \alpha \|$ for $x \in V$.

14. If E is an orthogonal projection of V onto W, prove that $\alpha - E\alpha \in W^{\perp}$ for all $x \in V$.

(14 × 1 = 14 weightage

Part B (Paragraph Type)

Answer any **seven** *questions.*

Each question has weightage 2.

15. Prove that $(1,2,3) \in \mathbb{R}^3$ is a linear combination of $\alpha = (1,2,1)$ and $\beta = (1,2,2)$.

16. Verify whether $S = \{(x, x+1) : x \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.

17. If W_1, W_2 are subspaces of a vector space V, prove that $W_1 \cap W_2$ is a subspace of V.

18. Let V be a vector space of dimension n. Prove that any set of $n + 1$ vectors of V is linearl dependent.

19. Find the matrix of the transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ given by $T(x,y,z) = (x+y, x+z, y+z)$ relativ to the ordered basis $B = \{(1,1,0),(0,1,1),(1,0,1)\}$.

20. Let $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be a basis of a vector space V and $\{f_1, f_2, \ldots, f_n\}$ be the dual basis of V

Prove that $f = \sum_{i=1}^{n} f(\alpha_i) f_i$ for each $f \in V^*$.

21. Show that similar matrices have same characteristic polynomial.

22. Express $\mathbb{R}^2$ as a direct sum of two one-dimensional subspaces.

23. Let T be a linear operator on a vector space V and let $V = W_1 \oplus \ldots \oplus W_k$, where each W_i invariant under T. Prove that if each W_i is one-dimensional then T is diagonalizable.

24. Verify whether $(x \mid y)$ defined as $(x \mid y) = x_1 + y_1$ is an inner product for :

$x = (x_1, x_2),\ y = (y_1, y_2) \in \mathbb{R}^2$.

(7 × 2 = 14 weightag

Part C (Essay Type)

Answer any **two** *questions.*
Each question has weightage 4.

25. (a) Define linearly independent set in a vector space.

    (b) Let A be an $n \times n$ matrix over a field F. Prove that if the row vectors of A form a linearly independent set then A is invertible.

26. Let V be a finite dimensional vector space and $T : V \rightarrow V$ be a linear operator. Prove that the following are equivalent :

    (i) T is invertible.

    (ii) T is one-to-one.

    (iii) T is onto.

27. (a) Define the annihilator W^0 of a subspace W of a vector space V.

    (b) Show that if V is finite dimensional then $\dim W + \dim W^0 = \dim V$.

28. (a) Prove that an orthogonal set of non-zero vectors is linearly independent.

    (b) Let W be a subspace of an inner product space V and $\beta \in V$. Show that $\alpha \in W$ is a best approximation to β if and only if $\beta - \alpha \in W^{\perp}$.

(2 × 4 = 8 weightage)