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D 72913 (Pages : 2) Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Statistics

ST 1C 02—ANALYTICAL TOOLS FOR STATISTICS—I

(2013 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

(Answer **all** *questions. Weightage* 1 *for each question).*

1. State Taylor's theorem for a multivariable function.
2. Prove that if $a > 0$ $\int_0^\infty \frac{dx}{x^4 + a^4} = \frac{\pi\sqrt{2}}{4a^3}$.
3. Show that the function $f(x, y) = x^2 - 2xy + y^2 + x^4 + y^4$ has minimum at the origin.
4. What is meant by a harmonic function ?
5. Establish the Cauchy Riemann conditions for an analytic function.
6. State Poisson Integral formula.
7. If a function is analytic show that it is independent of $\bar{z}$.
8. Define the terms pole and essential singularities giving *one* example each.
9. Distinguish between the terms residue at a pole and residue at infinity.
10. Define the inverse Laplace transform of a function.
11. State Fourier integral theorem.
12. Define Fourier transform of a function.

Part B

(Answer any **eight** *questions. Weightage* 2 *for each question).*

13. Define continuity of a function of two variables. Show that $f(x, y) = \frac{xy^2}{x^2 + y^6}$, $x, y \neq 0$, and $f(0, 0) = 0$ is not together continous at $(0, 0)$ while it is continous in x and y seperately.
14. Find the minimum value of $f(x, y) = x^2 + 5y^2 - 6x + 10y + 6$.

15. State and prove Morera's theorem.

16. Establish Jordan's lemma.

17. State and prove Liouvulle's theorem.

18. Obtain the harmonic conjugate of $U(x, y) = 2x - x^3 + 3xy^2$.

19. Find the Laplace transform of the following

    (i) $sin^3 6t$ (ii) $sin3t\ cos2t$.

20. Find the inverse Laplace transform of $\frac{3s+7}{s^2-2s-3}$.

21. State and prove the properties of Fourier Transforms.

22. Find the minimum of $f(x, y, z) = -x^2 - 2y^2 - z^2 + xy + z$ subject to $x + y + z = 35$.

23. Define Laplace transform of a function and state its properties.

24. Find the Laurent's series expansion for $f(z) = \frac{1}{z^2(1-z)}$ and specify the region in which t

    expansion is valid.

Part C

(Answer any **two** *questions. Weightage* 4 *for each question).*

25. (a) State and prove the Convolution Theorem for Laplace Transforms.

    (b) If $L[F(t)] = f(s)$ then show that $L\left[\int_0^t F(u)\,du\right] = \frac{f(s)}{s}$.

26. State and prove Taylor's theorem and expand $\frac{1}{z}$ about the point $z = 1$ using Taylor's theore

27. Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 2, \quad y(0) = 0, y'(0) = 1$ by the method of Laplace transforms.

28. Show that $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \frac{2\pi}{\sqrt{3}}$.