D 72914 (Pages : 2) Name.............58.............

Reg. No..............................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Statistics

ST 1C 03—ANALYTICAL TOOLS FOR STATISTICS—II

(2013 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** *questions. Weightage* 1 *for each question.*

1. Define inner product of vectors.
2. Define a vector subspace.
3. Define dimension of a vector subspace.
4. If A and B are symmetric matrices, then show that AB + BA is a symmetric matrix.
5. Define idempotent and Hermitian matrices.
6. Define rank of a matrix, show that rank of the product of two matrices cannot exceed the rank of either matrix.
7. Define characteristic roots and characteristic vectors of matrix A.
8. Show that the matrices AB and BA have same characteristic roots.
9. Show that the characteristic roots of a real symmetric matrix are real.
10. If $\lambda_1, \lambda_2, \ldots \lambda_m$ are the characteristic roots of a square matrix A. Then show that $|A| = \prod_{i=1}^{m} \lambda_i$.
11. State spectral decomposition theorem for real symmetric matrices.
12. Define generalized inverse of matrix and show that every matrix has a *g*-inverse ?

(12 × 1 = 12 weightage)

Part B

Answer any **eight** *questions. Weightage* 2 *for each question.*

13. Examine whether or not the following sets is a basis $\{(5, 3, 7), (1, -3, 6), (0, 3, 1)\}$ in R^4.
14. Let V be a vector space with dimension *n*. Show that any linearly independent set in V can be extended to a basis of V.
15. Show that every linearly independent set of vectors can be extended so as to constitute a basis of V_n.
16. Describe the method of finding inverse of a matrix by forming a partition of A.

Turn over

17. If A is an idempotent matrix of order m, show that,

(a) $I_m - A$ is also idempotent.

(b) Eigen values of A is 0 or 1.

18. Show that rank of an idempotent matrix is equal to trace of that matrix ?

19. Given $A = \begin{pmatrix} 1 & 4 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$. Find the inverse of the matrix A.

20. Find eigen values and eigen vectors of $\begin{pmatrix} 4 & -6 & 2 \\ 0 & 2 & -4 \\ 0 & 0 & 3 \end{pmatrix}$.

21. For a real symmetric matrix, show that characteristic vectors corresponding to distinct character roots are orthogonal.

22. Define the rank, signature and index of a real quadratic form. State the interrelationship betw them, if any ?

23. Classify the quadratic form $9x_1^2 + 4x_2^2 + 4x_3^2 + 8x_1 x_2 + 12x_1 x_3 + 12x_2 x_3$.

24. Given $A = \begin{pmatrix} 4 & 2 & 1 \\ -1 & 0 & 3 \\ 1 & 2 & 3 \end{pmatrix}$. Find g-inverse of A.

($8 \times 2 = 16$ weight

Part C

Answer any **two** *questions. Weightage 4 for each questions.*

25. Define a vector space, stating the axioms. Check whether the elementary n-vectors form a ve space.

26. (a) Show that the vectors $(2, 3, -1, -1), (1, -1, -2, -4), (3, 1, 3, -2), (6, 3, 0, -7)$ form a line dependent set. Also express one of these as a linear combination of the other.

(b) Show that the vectors (3, 1,2), (2, 1, 4) and (1, 1, 1) is a basis for the vector space in R^3.

27. (a) State and prove Cayley-Hamilton theorem.

(b) If $\lambda_1, \lambda_2, \ldots,$ are the characteristic roots of a square matrix A. Then show that t $(A' A) \geq \sum \lambda_i^2$.

28. (a) State and prove the necessary and sufficient condition that a real quadratic form X′A positive definite ?

(b) Determine the geometric and algebraic multiplicities of eigen values of the

matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$.

($2 \times 4 = 8$ weigh