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Name.....

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Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Statistics

ST IC 04—REGRESSION AND LINEAR PROGRAMMING

(2013 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer all questions. Weightage 1 each.

1. Prove that intersection of any two subspaces S and T of a vector space V is a subspace of V.
2. Examine the linear independence or dependence of the vectors $\{[3 \ 2 \ 4], [1 \ 0 \ 2], [1 \ -1 \ -1]\}$.
3. Define the feasible region of LPP and show that the feasible region is a convex set.
4. Solve the following problem graphically :

$$\text{Maximize } Z = 60 x_1 + 90 x_2$$

$$\text{subject to } x_1 + 2x_2 \leq 40$$

$$2x_1 + 3x_2 \leq 90$$

$$x_1 - x_2 \geq 10$$

$$x_1, x_2 \geq 0$$

5. Define feasible solution, basic feasible solution and non-degenerate basic feasible solution of a LPP.
6. Distinguish between degeneracy and cycling in a LPP. Give an example.
7. Explain the concept of duality and its uses in LPP.
8. Write a short note on post optimal sensitivity analysis.
9. What is integer programming ? Explain the merits and demerits of 'rounding off' a continuous optimal solution to a LPP to obtain an integer solution.
10. Define an assignment problem. Can it be considered as a particular case of a transportation problem ? If so, give reasons.
11. What is symmetric game ? Show that the value of a symmetric game is zero.
12. Explain travelling salesman problem.

(12 × 1 = 12 weightage)

Turn over

Part B

Answer any **eight** questions.

Weightage 2 each.

13. Examine whether the following set of vectors constitute a basis of V_3 $\{[1 - 1 0], [3 0 0], [0 2 1]\}$.
14. If the subspaces S_1, S_2, \dots, S_k are orthogonal to one another then $S_1 + S_2 + \dots + S_k$ is direct.
15. Define the inner product on a vector space V over F . In any inner product space, prove that

$$\langle x, \beta_1 y_1 + \beta_2 y_2 \rangle = \beta_1 \langle x, y_1 \rangle + \beta_2 \langle x, y_2 \rangle.$$

16. Explain the simplex procedure in solving an LPP.
17. Solve the following LPP using simplex method :

$$\text{Minimize } Z = 10x_1 + 6x_2 + 2x_3$$

$$\begin{aligned} \text{subject to } & -x_1 + x_2 + x_3 \geq 1 \\ & 3x_1 + x_2 - x_3 \geq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

18. Explain revised simplex method.
19. Prove that the dual of the dual of a given primal is again primal.
20. Use duality to solve the following LPP :

$$\text{Maximize } Z = 2x_1 + x_2$$

$$\begin{aligned} \text{subject to } & x_1 + 2x_2 \leq 10 \\ & x_1 + x_2 \leq 6 \\ & x_1 - x_2 \leq 2 \\ & x_1 - 2x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$

21. Explain any Gomory's method of solving an integer programming problem.
22. Prove that the number of basic variables in a transportation problem are almost $m + n - 1$, where m is number of origins and n is number of destinations.
23. Describe the Hungarian method of solving an assignment problem.
24. Explain the theory of dominance in the solution of rectangular games. Illustrate with example.

(8 × 2 = 16 marks)

Part C

Answer any **two** questions. Weightage 4 each.

25. Define basis of vector space. If $A \subseteq S$ where S is a subspace of a vector space V , then show the following statements are equivalent :

- (i) A is a minimal generating set of S.
- (ii) Every element of S can be expressed uniquely as a linear combination from A,
- (iii) A generates S and A is linearly independent.
- (iv) A is maximal linearly independent subset of S.

26. Use revised simplex method to solve the following LPP :

$$\text{Maximize } Z = 6x_1 - 2x_2 + 3x_3$$

$$\text{subject to } 2x_1 - x_2 + 2x_3 \leq 2$$

$$x_1 + 4x_3 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

27. Solve the following integer programming problem optimally :

$$\text{Maximize } Z = 8x_1 + 6x_2$$

$$\text{subject to } 8x_1 + 4x_2 \leq 85$$

$$3x_1 + 6x_2 \leq 95$$

$$x_1, x_2 \geq 0 \quad \text{and integers.}$$

28. Find the optimum solution to the following transportation problem :

		To						
		9	12	9	6	9	10	5
		7	3	7	7	5	5	6
From		6	5	9	11	3	11	2
		6	8	11	2	2	10	9
		4	4	6	2	4	2	22

(2 × 4 = 8 weightage)