15P105 Name : ....

Reg.No : ....

FIRST SEMESTER M.Sc. DEGREE EXTERNAL EXAMINATION FEB. 2016

(2015 Admission)

CC15P MT1 C05: DISCRETE MATHEMATICS

(Mathematics)

Time: Three Hours **Maximum: 36 Weightage**

Part A (Short Answer Questions)

Answer **all** Questions

Each question carries 1 Weightage

1. Define connectivity of a graph. Also find the connectivity of $K_{3,4}$
2. Define dual of a plane graph.
3. Give an example of an Eulerian graph with 5 vertices and 7 edges.
4. Define directed graph with an example.
5. Determine which complete bipartite graphs are complete graphs.
6. Give an example of a simple regular graph with 5 edges.
7. Define a partial ordering on a set X.
8. Define a Boolean algebra.
9. Define upper bound and lower bound on a partially ordered set.
10. Prove that every tree with at least two vertices has at least two leaves.
11. Define nfa.
12. State Jordan Curve Theorem.
13. Define atom with an example.
14. What are Regular languages?

(**14 x 1 = 14 Weightage)**

Part B (Short Essay Questions)

Answer any **seven** from the following ten questions (15 - 24)

Each question carries 2 Weightage

15. From the definition of isomorphism, prove that $G \cong H$ *if and only if* $\overline{G} \cong \overline{H}$.
16. Draw Petersen graph. Also show that Petersen graph is not bipartite.
17. If G is a simple graph, then K(G)≤ $K'(G)$ ≤δ(G).
18. State and prove Euler's formula.
19. Let X be a finite set and ≤ be a partial order on X . Also R is a relation on X defined by xRy if and only if y *covers* $x(w.r.t. \leq)$. Show that ≤ is generated by R.

20. Show that every finite Boolean algebra is isomorphic to a power set Boolean algebra, specifically, to the power set Boolean algebra of the set of all its atoms.
21. Write the following Boolean functions in their disjunctive normal forms:
    (i) $f(x_1, x_2, x_3) = (x_1 + x_2')x_3' + x_2x_1'(x_2 + x_1'x_3)$.
    (ii) $g(a, b, c) = (a + b + c)(a' + b + c')(a + b' + c')(a' + b' + c')(a + b + c')$.
22. Explain transition graph of a dfa with an example.
23. Show that $L = \{vwv: v, w \in \{a, b\}^*, |v| = 2\}$ is regular.
24. What do you mean by equivalent grammars? Explain with example.

(7 x 2 = 14 Weightage)

Part C (Essay Questions)

Answer any **two** from the following four questions (25-28)

Each question carries 4 weightage

25. Prove that the complete graph K_n can be expressed as the union of k bipartite graphs if and only if $n \leq 2^k$.
26. Show that for an n-vertex graph G (with $n \geq 1$), the following are equivalent.
    (i) G is connected and has no cycles.
    (ii) G is connected and has $n - 1$ edges.
    (iii) G has $n - 1$ edges and no cycles.
    (iv) G has no loops and has, for each $u, v \in V(G), exactly\ one\ u, v - path.$
27. Let $(X, +, \cdot\,,)$ be a Boolean algebra. Show that the following properties hold for all elements $x, y, z\ of\ X$.
    (i) $x + x = x\ and\ x \cdot x = x$
    (ii) $x + 1 = 1\ and\ x \cdot 0 = 0$
    (iii) $x + x \cdot y = x\ and\ x \cdot (x + y) = x$
    (iv) $x + (y + z) = (x + y) + z\ and\ x \cdot (y \cdot z) = (x \cdot y) \cdot z$
28. Show that the grammar G with $\sum = \{a, b\}$ and productions $S \to SS, S \to \lambda, S \to aSb, S \to bSa$ generates the language L={w:$n_a(w) = n_b(w)$}.

(2 x 4 = 8 Weightage)

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