16P157 (Pages:3) Name: ……………………….

Reg. No……………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P ST1 C05 – DISTRIBUTION THEORY

(Statistics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer **all** questions.

Each question carries a weightage of 1

1. If X represents the number of failures preceeding the first success in a Bernoulian trial, identify the distribution of X. For this distribution, show that the relationship $P(X > m + n|X > m) = P(X \geq n)$
2. Identify a discrete distribution for which mean = variance. State the reproductive property for this distribution.
3. If x and Y are two independent binomial random variable. Then show that conditional distribution of X given (X+Y) is hypergeometric.
4. The joint probability density function of r.v's X and Y are given by $P(X = 1, Y = -1) = 1/3, P(X = 0, Y = 1) = 1/3$ *and* $P(X = 1, Y = 1) = 1/3$. Find the marginal distribution of X and Y.
5. Define exponential family of distributions. Identify two members of the family.
6. Define lognormal distribution. Obtain the distribution of (1/x).
7. Define non-central t distribution. When will this reduce to central t?
8. If Y is a r.v following the classical pareto distribution, obtain the distribution of $X = \log Y$.
9. If X has an F distribution with (m_1, m_2) degrees of freedom, obtain the distribution of (1/x).
10. Given $f(x, y) = 2, \quad 0 < x < y; \quad 0 < y < 1$. Evaluate the conditional expectation $E(Y|X = x)$.
11. Give two examples each for continuous distributions which are (a)Symmetric and (b) Skewed.
12. Find the m.g.f of normal distribution.

(12×1= 12 weightage)

Part B

Answer any **eight** questions.
Each question carries a weightage of 2.

13. Define probability generating function associated with a r.v. When will this reduce to (i) characteristic function (ii) Moment generating function.If P(s) is the probability generating function associated with a non-negative integer valued r.v, show that

$$\sum_{n=0}^{\infty} s^n P(X \leq n) = \frac{P(s)}{1-s}$$

14. Obtain the characteristic function of the Cauchy distribution. Show that then arithmetic mean $\overline{X}$ of a sample X_1,X_2,....,X_n of independent observations from a Cauchy distribution is also a Cauchy variate.

15. (x,y) follow a trinomial distribution with p.m.f :

$$f(x,y) = \frac{P(n!)}{x!y!(n-x-y)!} p_1^x p_2^y p_3^{n-x-y}$$

where x and y are non-negative integers such that x+y $\leq$ n and p_1,p_2,p_3 >0 with p_1+p_2+p_3 $= 1$.Find $E(y|X=x)$.

16. Explain Weibull distribution. Describe how it can be derived by transformation from an exponential r.v. Obtain the characteristic function and deduce the mean and variance.

17. Define bivariate distribution. Also find the conditional distribution of bivariate distribution.

18. Define non-central Chi-square statistic and write down its p.d.f. Deduce the central chi-square. Also write down two applications of Chi-squre distribution.

19. Define a finite mixture of probability density function. Verify that a mixture of p.d.f's satisfies the properties of a p.d.f. If the associated r.v's are normally distributed, obtain the distribution of the mixture distribution.

20. The joint.p.d.fof (X,Y) is given by $f(x,y) = xe^{-x(1+y)}, x \geq 0, y > 0$.Find the marginal distribution of Y and show that E(Y)does not exist.Evaluate the conditional expectation $E(Y|x)$

21. Define Pearson family of distributions. Show that gamma and beta distributions are members of this family.

22. Show that exponential distribution is a special case of Gamma distribution. If X_1,X_2.....X_n are iid r.v's following the exponential distribution, obtain the distribution of Min (X_1,X_2.....X_n).

23. Define order statistic.Give an outline of the steps involved in evaluating the distribution of sample range. Illustrate with example.

24. Find the sampling distribution of the sample mean $\overline{X}$ if X follows Chi-square distribution with n d.f

(8×2= 16 weightage)

Part C

Answer any **two** questions.
Each question carries a weightage of 4.

25. Define power series family. Identify three members of the family. Obtain the m.g.f of the distribution and deduce the mean and variance. Also obtain a recurrence relation satisfied by the cumulants.

26. Show that :
(i) If X follow the uniform distribution with $F(x) = x, 0 \leq x \leq 1$, then $Y = -logX$ is unit exponential.
(ii) If X the Weibull distribution with $F(x) = 1 - exp(-x^a)$, $x \geq 0$, $a > 0$
(iii) If X follow the Pareto law with $F(x) = 1 - x^a$, $a > 0$, $x \geq 1$, then W = a log X follow unit exponential.
(iv) if X follow the logistic distribution with $F(x) = (1 + e^{-ax})^{-1}$, $a > 0$, x is real then $Y = log(1 + e^{-ax})$ is unit exponential.

27. (i) If $X_{r:n}$, $r = 1, 2, \ldots.., n$ are the order statistics of a random sample of size of n drawn from an absolutely continuous distribution then obtain the conditional p.d.f of $X_{s:n}$ given $X_{r:n} = x$ and $X_{t:n} = y$ for $1 \leq r < s < t \leq n$.
(ii)State Chebyshev's inequality. If X be distributed with p.d.f f(x)=1 for $0 < x < 1$ and equal to zero otherwise.Prove that
$p[|X - \frac{1}{2}| < 2\sqrt{\frac{1}{12}}] \geq 0.75$

28. Define non-central Chi-square distribution and derive its p.d.f. Also find the expression of mean and variance.

(2×4= 8 weightage)

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