15P154 **Name:……………..………….**

Reg No:………………………

FIRST SEMESTER M.Sc. DEGREE EXTERNAL EXAMINATION FEBRUARY 2016

(2015 Admission)

CC15P ST1 C02– Analytical Tools for Statistics 1

(STATISTICS)

Time : 3 Hrs **Maximum: 36 Weightage**

Part A (Answer *all* questions)

Define Riemann integral of a multivariable function.

1. Let $f(x,y) = \begin{cases} \frac{xy}{x^2-y^2}, & if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0) \end{cases}$

   Show that this function possess partial derivatives at (0,0).
2. Obtain the necessary condition for a function $f(z)$ to be analytic.
3. If f is analytic in a domain S and if $|f|$ is constant there, then show that f is constant.
4. Show that $f(z) = x - iy^2$ is differentiable only at $y = -\frac{1}{2}$ and $f'(z) = 1$
5. Define pole of order *m* of a function $f(z)$
6. What is removable singularity?
7. If $\mathcal{L}\{F(t)\} = f(s)$, then find $\mathcal{L}\{t^n F(t))\}$
8. State the maximum modulus principle.
9. Define the inverse Laplace Transform of a function
10. Define half range Fourier sine and cosine series.
11. State the convolution theorem for Fourier transforms.

(12 x 1=12 weightage)

Part B (Answer any eight questions. Weightage 2 for each question)

12. Define partial derivatives. Show that

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, (x,y) \neq (0,0) \\ 0, \ (x,y) = (0,0) \end{cases}$$

   is not continuous at the origin and the partial derivatives of $f(x,y)$ with respect to x and y exist at the origin.
13. Discuss the maxima and minima for $f(x,y) = 2x^4 + y^4 - 2x^2 - 2y^2$
14. State and prove Morera's theorem.
15. Show that the function $f(z) = x^3 + 3xy^2 + i(y^3 + 3x^2y)$ is differentiable only at points that lie on the coordinate axes.

16. Show that the function $v(x, y) = Cosx\,.Cosh\,y$ is harmonic and find the corresponding analytic function.
17. Find the Laurent series for the function $\frac{(z^2-2z+7)}{(z-2)}$ in the domain $|z-1| > 1$
18. Evaluate $\int_0^1 (1 + it^2)dt$
19. Establish Jordan's lemma.
20. Show that $\int_0^\infty \frac{(\log x)^2}{1+x^2}\,dx = \frac{\pi^2}{8}$
21. Find the Laplace transform of : (i) $e^{3x}\cos 2x$ (ii) $\sin^3 6x$
22. Fund the inverse Laplace transform of $\frac{e^{-5\,s}}{(s-4)^4}$
23. Find the Fourier transform of

$$f(x) = \begin{cases} x\ , & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

(8 x 2=16 weightage)

Part C (Answer any two questions. Weightage 4 for each question)

24. State and prove Laurent series expansion
25. State and prove Cauchy Goursat theorem.
26. Solve the differential equation by the method of Laplace transform:

$$tY'' + (1-2t)Y' - 2Y = 0,\ Y(0) = 1,\ Y'(0) = 2\ .$$

27. By Contour integration prove that :

$$\int_0^\infty \frac{sin^2x}{x^2}\,dx = \frac{\pi}{2}$$

(2 x 4=8 weightage)

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