15P102 **Name: …………………………..**

Reg. No…………………………

FIRST SEMESTER M.Sc. DEGREE EXTERNAL EXAMINATION FEBRUARY 2016

(2015 Admission)

CC15P MT1 C02 – LINEAR ALGEBRA

(Mathematics)

Time: 3 hrs **Max. Weight: 36**

PART A (Short Answer Type)
Answer *all* questions.
Each question has weightage 1.

1. Let V be a vector space over the field F. Prove that $(-c)\alpha = -c\alpha$ for $c \in F$ and $\alpha \in V$.
2. Prove that the vectors $(1,2,1)$ and $(1,0,1)$ are linearly independent in $\mathcal{R}^3$.
3. Find the dimension of the space of $3x3$ symmetric matrices over the field of real numbers.
4. Give two linear functionals f_1, f_2 on $\mathcal{R}^2$ such that $\{f_1, f_2\}$ is linearly independent.
5. Let $T: \mathcal{R}^2 \to \mathcal{R}^2$ be defined by $T(x,y) = (x+2y, 3x-y)$. Find the matrix of T relative to the standard basis of $\mathcal{R}^2$.
6. Find the null space of the transformation defined by $T(x,y) = (x+y, x)$.
7. Find a characteristic vector of the operator $T: \mathcal{R}^2 \to \mathcal{R}^2$ defined by $T(x,y) = (x+y, x-y)$.
8. If T is any linear operator on V, then prove that the zero subspace invariant under T.
9. Define similar matrices.
10. Prove that similar matrices have the same characteristic polynomial.
11. If E_1 and E_2 are projections onto independent subspaces, then $E_1 + E_2$ is a projection. True or false. Why?
12. Find a $3x3$ matrix for which the minimal polynomial is x^2.
13. If S is any subset of an inner product space V then prove that its orthogonal complement $S^{\perp}$ is always a subspace of V.
14. With respect to the standard inner product, write an orthogonal vector of $(-2,-4,1)$ in $\mathcal{R}^3$. **(14 x 1 = 14 Weightage)**

PART B (Paragraph type)
Answer any *seven*
Each question has weightage 2.

15. Let W be the subset of V, the vector space of all $2x2$ matrices over a field F, of the form $\begin{bmatrix} x & y \\ z & 0 \end{bmatrix}$ where x, y, z are arbitrary scalars in F. Prove that W is a subspace of V and find $dim\, W$.
16. Find the coordinates of the vector $(a,b,c) \in \mathcal{R}^3$ relative to the ordered basis $\{(1,0,-1), (1,1,1), (1,0,0)\}$.

17. Prove that the intersection of any collection of subspaces of a vector space V is a subspace of V.
18. Prove that a linear transformation is one-to-one if and only if its null space is zero.
19. Let $\mathfrak{B} = \{(1,1), (1,0)\}$ and $\mathfrak{B}' = \{(0,1), (1,1)\}$ be two ordered bases of $\mathcal{R}^2$. Then find the $2x2$ matrix P with entries in $\mathcal{R}$ such that $[\alpha]_{\mathfrak{B}} = P[\alpha]_{\mathfrak{B}'}$.
20. Let $p(x)$ is the minimal polynomial of a linear operator T. Show that $p(c) = 0$ if and only if c is a characteristic value of T.
21. Let T be a linear operator on a vector space V and let $\alpha \in V, c \in F$ be such that $T(\alpha) = c\alpha$. Show that for any polynomial f, $f(T)\alpha = f(c)\alpha$.
22. Let W_1, W_2 subspaces of a vector space V and let $V = W_1 \oplus W_2$. Show that there is a projection on V with range W_1 and null space W_2.
23. In an inner product space, prove that an orthogonal set of non-zero vectors is linearly independent.
24. State and prove Bessel's inequality in an inner product space.

(7 x 2 = 14 Weightage)

PART C (Essay type)
Answer any *Two*
Each question has weightage 4.

25. Describe the linear algebra $L(V)$ of linear operators on a vector space V. Prove that if dimension of V is n then dimension of $L(V)$ is n^2.
26. Define the transpose T^t of a linear operator T. Prove that
    (i) null space of T^t is the annihilator of the range of T.
    (ii) rank (T^t) = rank (T).
27. Let T be a linear operator on a finite-dimensional space V. Let $c_1, c_{2,} \ldots .. c_k$ be the distinct characteristic values of T and W_i be the null space of $(T - c_i I)$. Prove the following are equivalent.
    (i) T is diagonalizable.
    (ii)The characteristic polynomial for T is $f = (x - c_1)^{d_1}(x - c_2)^{d_2} \ldots (x - c_k)^{d_k}$ and $dimW_i = d_i, i = 1,2, \ldots ., k$.
    (iii) $\dim(W_1) + \dim(W_2) + \cdots + dim(W_k) = dimV$.
28. State and prove Gram-Schmidt orthogonalization process in an inner product space. Apply this process to the vectors $(1,0,1), (1,0,-1), (0,3,4)$ to obtain an orthonormal basis for $\mathcal{R}^3$ with the standard inner product.

(2 x 4 = 8 Weightage)

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