16P102 (Pages:2) Name: ……………………….

Reg. No……………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C02 – LINEAR ALGEBRA

(Mathematics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

PART A

(Short Answer Type)

Answer *all* questions.

Each question has weightage 1.

1. If V be a vector space over the field F, prove that $(-1)\alpha = -\alpha$, where $-1 \in F$ and $\alpha \in V$.
2. Write a basis of the vector space of all 3x3 diagonal matrices over the field $\mathbb{R}$.
3. Verify the function $T(x,y) = (x+y, 2x+1)$ from $\mathbb{R}^2$ into $\mathbb{R}^2$ linear?
4. Let T be the linear operator on $\mathbb{R}^2$ defined by $T(x,y) = (x-y, 2x+y)$. What is the matrix of T relative to the standard ordered basis?
5. Give a linear functional on $\mathbb{R}^2$.
6. Define hyperspace of a vector space.
7. Define the transpose of a linear transformation.
8. Let T be the linear operator on $\mathbb{R}^2$ which is represented in the standard ordered basis by the matrix $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. Prove that the only subspaces of $\mathbb{R}^2$ which are invariant under T are $\mathbb{R}^2$ and the zero subspace.
9. Show that every matrix A such that $A^2 = A$ is similar to a diagonal matrix.
10. Prove that similar matrices have the same characteristic polynomial.
11. Let V be a vector space and $(|)$ be an inner product on V. Show that if $(\alpha|\beta) = 0$ for all $\beta \in V$, then $\alpha = 0$.
12. If V is an inner product space, then prove that $\|c\alpha\| = |c|\|\alpha\|$ for any vector α in V and for any scalar c.
13. If S is any subset of an inner product space V then prove that its orthogonal complement $S^{\perp}$ is always a subspace of V.
14. With respect to the standard inner product, write an orthogonal vector of $(-2,-4,0)$ in $\mathbb{R}^3$.

(14 x 1 = 14 Weightage)

PART B

(Paragraph type)

Answer any *seven*

Each question has weightage 2.

15. Let V be a vector space of dimension n. Prove that every subset of V containing more than n elements is linearly dependent.
16. Find the range, rank, null space, and nullity for the zero transformation and the identity transformation on a finite-dimensional space V.
17. Prove that if W_1, W_2 are subspaces of a vector space V, then $W_1 + W_2$ is also a subspace of V.
18. Prove that a linear transformation is one-to-one if and only if its null space is zero.
19. Let T be a linear operator on a finite dimensional vector space V. Prove that if T maps a basis of V to a basis of V then T is invertible.
20. Let $p(x)$ is the minimal polynomial of a linear operator T. Show that $p(c) = 0$ if and only if c is a characteristic value of T.
21. If S is any subset of a finite-dimensional vector space V, then $(S^{\circ})^{\circ}$ is a subspace spanned by S.
22. Let E be a projection on a vector space V and R be the range and N be the null space of E. Prove that $R \oplus N = V$.
23. In an inner product space, prove that an orthogonal set of non-zero vectors is linearly independent.
24. State and prove Cauchy-Schwarz inequality in an inner product space.

(7 x 2 = 14 Weightage)

PART C

(Essay type)

Answer any *Two*

Each question has weightage 4.

25. Let V be a n-dimensional vector space over the field F, and $\mathfrak{B}$ and $\mathfrak{B}'$ be two ordered bases of V. Then prove that there is a unique invertible $n \times n$ matrix P such that $(i)[\alpha]_{\mathfrak{B}} = P[\alpha]_{\mathfrak{B}'}$ $(ii)[\alpha]_{\mathfrak{B}'} = P^{-1}[\alpha]_{\mathfrak{B}}$.
26. Let V be a finite dimensional vector space over the field F, and let W be a subspace of V. Prove that $dim\, W + dim\, W^{\circ} = dim\, V$ and if W is a k-dimensional subspace of an n-dimensional vector space V, then W is the intersection of $(n - k)$ hyperspaces in V.
27. Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.
28. Let W be a finite-dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then prove that E is an idempotent linear transformation of V onto W, $W^{\perp}$ is the null space of E, and $V = W \oplus W^{\perp}$.

(2 x 4 = 8 Weightage)

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