**16P107** (Pages:2) Name: …………......

Reg. No…………....

# FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(Regular/Supplementary/Improvement)

(CUCSS-PG)

## CC15P PHY1 C02 – MATHEMATICAL PHYSICS - I

(Physics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

### Section A

Answer **all** questions, each question carries **1** weightage.

1. Show that angular velocity of rotation of a rigid body is half the curl of a velocity vector within the body.
2. What are characteristics of orthogonal curvilinear coordinates?
3. Show that eigen values of a Hermitian matrix are real and eigen vectors belonging to different eigen values are orthogonal.
4. What do you mean by contraction of a tensor? Illustrate with an example.
5. What are the different forms of boundary conditions employed while solving differential equations?
6. Show that a general second order homogeneous ordinary differential equation has only two linearly independent solutions.
7. Define Hermitian operator. Write any two properties of Hermitian operator.
8. What is meant by a self adjoint ordinary differential equation? Explain with example.
9. Prove that $P_n(1) = 1$ and $P_n(-1) = (-1)^n$
10. Define Laplace transform of a function. State and prove the first shifting property of Laplace transform.
11. Evaluate $\int_0^\infty \cos x^2 dx$ .
12. Define Dirac delta function and show that it is the derivative of unit step function.

(12 × 1 = 12 weightage)

## Section B

Answer any **two** questions. Each question carries **6** weightage.

13. Derive the expression for gradient, divergence and curl in general curvilinear coordinates. Reduce the curl in spherical polar coordinates.
14. Obtain the series solution of Legendre equation by the method of Frobenius and get an expression for Legendre polynomial.
15. (a) Using the generating function for Bessel function, derive the basic recurrence relations.

    (b) Using Laplace transform solve the equation $\frac{d^2x}{dt^2} + \omega^2 x = 0$ for the initial conditions $x(0) = x_0$ and $x'(0) = 0$.
16. Explain Gram-Schmidt orthogonalisation procedure with suitable example.

(2 × 6 = 12 weightage)

## Section C

Answer any **four** questions. Each question carries **3** weightage.

17. Show that $J_0(x) - 2J_2(x) + 2J_4(x) + \ldots = \cos x$ .
18. Show that $\int_{-1}^{1} xP_n(x)P_{n-1}(x)dx = \frac{2n}{4n^2-1}$ .
19. If $f(t) = t^a$ and $g(t) = t^b$, show that the convolution

    $f * g = L^{-1}\{F(s)G(s)\} = t^{a+b+1} \int_0^1 y^a(1-y)^b dy.$

    Hence show that $\int_0^1 y^a(1-y)^b dy = \frac{a!b!}{(a+b+1)!}$
20. Represent the function $f(x) = x$, $-\pi < x < \pi$ in the form of Fourier series and show that

    $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots = \frac{\pi}{4}$
21. Define covariant and contra-variant tensors. Show that Kronecker delta can be expressed as a mixed tensor.
22. Given that $A = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$. Show that $(I-A)(I+A)^{-1}$ is a unitary matrix.

(4 × 3 = 12 weightage)

*******