15P104 Name………………

Reg. No………………

FIRST SEMESTER M.Sc. DEGREE EXTERNAL EXAMINATION FEBRUARY 2016

(2015 Admission)

CC15P MT1 C04 – ODE and Calculus of Variations

(Mathematics)

Time: 3hrs **Max. 36 Weightage**

Part A

Answer **All** Questions

Each Question carries 1 weightage

1. Locate and classify the singular points on the X axis of the differential equations $x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$.
2. Find indicial equation and its roots for $x^3y'' + (\cos 2x - 1)\, y' + 2xy = 0$.
3. Define Hyper geometric series $F(a,b,c,x)$ and show that $e^x = lim_{b\to\infty} F(a,b,a,\frac{x}{b})$.
4. Express a third degree polynomial in terms of Legendre polynomials.
5. Define gamma function and prove that $\Gamma(p+1) = p\Gamma p$.
6. Show that $\frac{d(J_0(x))}{dx} = -J_1(x)$.
7. Describe the phase portrait of $\frac{dx}{dt} = -x$
$\frac{dy}{dt} = -y$
8. Determine whether the function is positive definite, negative definite or neither:
$$f(x,y) = -x^2 - 4xy - 5y^2$$
9. State Sturm comparison theorem.
10. Show that $f(x,y) = y^{\frac{1}{2}}$ does not satisfy Lipschitz condition on the rectangle $|x| \le 1$ and $0 \le |y| \le 1$
11. Find the normal form of Bessel equation $x^2y'' + xy' + (x^2 - p^2)y = 0$.
12. Find the extremal for the integral $\int_{x_1}^{x_2} f(x,y,y')dx$ if the integrand is $\frac{\sqrt{1+(y')^2)}}{y}$.
13. Find the general solution of $\frac{dx}{dt} = 2x$
$\frac{dy}{dt} = 3y$
14. State Picard's theorem.

(14 x 1 = 14 Weightage)

Part B

Answer **any 7** questions

Each question carries 2 weightage

15. Solve $y' = 1 + y^2, y(0) = 0$ in two ways and show that $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \cdots$
16. Find a Frobenius series solution of the Bessel's equation of order zero.
17. Find the first three terms of Legendre series for the function $f(x) = \begin{cases} 0 & if -1 \le x < 0 \\ x & if\ 0 \le x \le 1 \end{cases}$

18. State Bessel expansion theorem and find the Bessel series for the function $f(x) = 1$.

19. Find the general solution of $\frac{dx}{dt} = 4x - 2y$

$$\frac{dy}{dt} = 5x + 2y$$

20. Define Liapunov function for an autonomous system and show that if there exist a Liapunov function $E(x, y)$ for the system then the critical point (0, 0) is stable.

21. Determine the nature and stability properties of the critical point (0, 0) for $\frac{dx}{dt} = -x - 2y$

$$\frac{dy}{dt} = 4x - 5y$$

22. If $q(x) < 0$ and $u(x)$ is a non-trivial solution of $u'' + q(x)u = 0$ then Prove that $u(x)$ has at most one zero.
23. Find the exact solution of the initial value problem $y' = 2x(1 + y), y(0) = 0$. Starting with $y_0(x) = 0$, calculate $y_1(x), y_2(x)$ and $y_3(x)$ using Picard's method.
24. Find the curve of fixed length L that joins (0, 0) and (1, 0) which lies above the X axis and encloses maximum area between itself and X axis.

(7 x 2 = 14 Weightage)

Part C

Answer **any 2** questions

Each question carries 4 weightage

25. State and prove orthogonalityproperty of Legendre polynomials.
26. Find the two independent Frobenius series solution for $xy'' + 2y' + xy = 0$.

27. For the non-linear system $\frac{dx}{dt} = y(x^2 + 1)$

$$\frac{dy}{dt} = -x(x^2 + 1)$$

    a. Find the critical point.
    b. Find the differential equation of the paths.
    c. Solve this equation to find the paths.
    d. Sketch a few paths and show the direction of increasing t.
28. State and prove Sturm separation theorem and show that the zeros of the function $a \sin x + b \cos x$ and $c \sin x + d \cos x$ are distinct and occur alternately whenever $ad - bc \neq 0$.

(2 x 4 = 8 Weightage)

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