**17P107** (Pages:2) Name: ……………………

Reg. No……………………..

**FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017**

(CUCSS-PG)

**CC17P MT1 C05 – DISCRETE MATHEMATICS**

(Mathematics)

(Regular 2017 Admissions)

Time: Three Hours Maximum: 36 Weightage

**Part A**

Answer ***all*** questions. Each question carries 1 weightage.

1. Define strict partial order and give an example of it. If $R$ is a partial order on a set $X$, then prove that $R - \{ (x,x): x \in X \}$ is a strict partial order on $X$.
2. Let $(X, +, \cdot, ')$ be a Boolean algebra. Show that $(x + 1) = 1$.
3. Define distributive lattice. Give an example of a lattice that is not distributive.
4. Distinguish between atom and coatom .
5. Prove that $e = xy$ is a cut edge of a connected graph $G$ if and only if there exist vertices $u$ *and* $v$ such that $e$ belongs to every $u - v$ path in $G$.
6. Define normal product in graphs.
7. Prove that $\lambda(K_n) = n - 1$.
8. Prove or disprove: If $H$ is a subgraph of $G$ then $\lambda(H) \leq \lambda(G)$.
9. Define Eulerian graph. Give an example of a graph that is not Eulerian.
10. Give an example of a non simple disconnected graph with $\delta \geq \frac{n-1}{2}$.
11. Describe the language generated by the grammar with productions, $S \to aSb, A \to \lambda$.
12. Define non-deterministic acceptor and give an example of it.
13. Define extended transition function $\delta^*$ with an example.
14. Let $u$ be a string on the alphabet $\Sigma$ . Prove that $|u^n| = n\,|u|$ for all $n = 1,2,3, \ldots$

**(14 × 1 = 14 Weightage)**

**Part B**

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Prove that the set $\Gamma(G)$ of all automorphisms of a simple graph $G$ is a group with respect to the composition of mappings as the group operation.
16. Prove that A connected graph $G$ with at least two vertices contains at least two vertices that are not cut vertices.
17. Show that the connectivity and edge connectivity of a simple cubic graph $G$ are equal.

18. If $d = (d_1, d_2, d_3, ..., d_n)$ is any sequence of nonnegative integers with $\sum_{i=1}^{n} d_i$ even, show that there exists a graph (not necessarily simple) with $d$ as its degree sequence.

19. Draw the Hasse Diagram for the lattice$(D_{20}, \leq)$. $D_{20}$ be the set of all divisors of 20 and $\leq$ be the relation 'divides'.

20. Prove that the characteristic numbers of a symmetric Boolean function completely determine it.

21. Define a subalgebra. Show that a subalgebra $Y$of a Boolean algebra $X$ is itself a Boolean algebra.

22. Find a dfa for the language $L = \{w: |w|\ mod\ 3 = 0\}$ on $\Sigma = \{a, b\}$.

23. Find a grammar that generates $L = \{a^n b^{2n}: n \geq 0\}$ .

24. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a dfa, and let $G_M$ be its associated transition graph. Then for every $q_i, q_j \in Q$, and $w \in \Sigma^+$, prove that $\delta^*(q_i, w) = q_j$ if and only if there is in $G_M$ a walk with label $w$ from $q_i$ to $q_j$.

**$(7 \times 2 = 14$ Weightage)**

**Part C**

Answer any ***two*** questions. Each question carries 4 weightage.

25. (a). Prove that a graph $G$ with at least three vertices is 2-connected if, and only if, any two vertices of $G$ are connected by at least two internally disjoint paths.

    (b). Prove that every connected graph contains a spanning tree.

26. (a). State and prove Euler's formula for a connected plane graph $G$.

    (b). Prove that if $G$ is a simple planar graph with at least 3 vertices, then $m \leq 3n - 6$.

    (c). Prove that the Petersen graph is nonplanar.

27. Distinguish between D. N. F. and C. N. F. Express the function $f(x_1, x_2, x_3) = x_1' x_2 (x_1' + x_2 + x_1 x_3)$ in its C.N.F. and D.N.F.

28. Define regular languages. Show that the language $L = \{vwv: v, w \in \{a, b\}^*, |v| = 2\}$ is regular.

**$(2 \times 4 = 8$ Weightage)**

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