17P160 (Pages:2) Name: ……………………

Reg. No……………………

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15PST1C03 – ANALYTICAL TOOLS FOR STATISTICS-II

(Statistics)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** the questions. Weightage 1 for each question.

1. Define algebraic and geometric multiplicities.
2. Show that the vectors $(1,\ 1,\ 1)$, $(3,\ 1,\ 2)$ and $(2,\ 1,\ 4)$ are linearly independent?
3. Define minimal polynomial and characteristic polynomial.
4. Show that for an idempotent matrix A, $\text{Rank(A)} = \text{Trace(A)}$?
5. What do you mean by Jordan canonical form?
6. Define (i) a quadratic form and (ii) index, rank and signature of a quadratic form.
7. What do you mean by generalized inverse of a matrix?
8. Define Similar matrices. Give an example.
9. What is rank factorization of a matrix?
10. Prove that every non-singular matrix is a product of elementary matrices.
11. Show that the two matrices A and $P^{-1}AP$ have the same characteristic roots, where P is any non-singular matrix.
12. Show that the form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_2x_3 - 2x_3x_1 + 2x_1x_2$ is an indefinite quadratic form.

$(12 \times 1 = 12$ weightage)

Part B

Answer any ***eight*** questions. Weightage 2 for each question.

13. If A and B are two matrices of order n and $\text{Rank(A)} = r$, $\text{Rank(B)} = s$, show that $r + s - n \leq \text{Rank (AB)} \leq \min(r, s)$
14. State and prove Cayley–Hamilton theorem.
15. Given $A = \begin{bmatrix} 1 & 4 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Find inverse of matrix A.
16. Define Moore–Penrose inverse of a matrix. Show that it is unique?
17. Prove that the eigen vectors associated with distinct eigen values of a matrix are linearly independent.
18. If A is $m \times m$ idempotent matrix, then show that :

    (a) $I_m - A$ is also idempotent.

    (b) Each eigen value of A is 0 and 1.

19. Find eigen values and eigen vectors of $A = \begin{bmatrix} 8 & -4 & 6 \\ 10 & -6 & 6 \\ 8 & -8 & 10 \end{bmatrix}$.

20. Prove that two eigen vectors associated with two distinct eigen values of a matrix are orthogonal to each other.

21. If $\lambda_1, \lambda_2, \cdots, \lambda_n$ are the characteristic roots of a matrix A, show that trace $(A^2) = \sum_{i=1}^{n} \lambda_i^2$.

22. If A and B are square matrices of same order, show that both AB and BA will have the same characteristic roots.

23. Explain the spectral decomposition of a matrix.

24. Show that $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix}$ is diagonalizable over R and find P such that $P^{-1}AP$ is diagonal.

(8 × 2 = 16 weightage)

PART C

Answer any *two* questions. Weightage 4 for each question.

25. (a) Prove that the geometric multiplicity of a characteristic root cannot exceed algebraic multiplicity of the same.

(b) If A is an $n \times n$ matrix with eigen values $\lambda_1, \lambda_2, \cdots, \lambda_n$, then prove that :

I) $trace(A) = \sum_{i=1}^{n} \lambda_i$

II) $|A| = \prod_{i=1}^{n} \lambda_i$

26. (a) State and prove the necessary and sufficient condition that a real quadratic form X'AX is positive definite.

(b) Determine the geometric and algebraic multiplicities of eigen values of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$.

27. (a) State and prove the necessary and sufficient condition for the diagonalizability a square matrix.

(b) Prove that every non-zero nilpotent matrix is not diagonalizable.

28. (a) Reduce the following Matrix in to its normal form and hence find rank

$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}$.

(b) If A is a hermitian matrix, show that A is unitarily similar to a diagonal matrix.

(2 × 4 = 8 weightage)

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