17P103 (Pages:2) Name: ……………………

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C03/ CC17P MT1 C03 - REAL ANALYSIS – I

(Mathematics)

(2015 Admission onwards)

Time: Three hours Maximum: 36 Weightage

PART A

(Short Answer Questions)

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove or disprove: Every continuous function is an open mapping.
2. State intermediate value theorem. Is the converse true?
3. Let E be a subset of a metric space X. Show that $\bar{E}$ closed.
4. Show that if $f_1(x) \leq f_2(x) on\ [a, b]$, $then \int_a^b f_1\, d\alpha \leq \int_a^b f_2\, d\alpha$
5. Discuss the differentiability of the modulus function in R.
6. Define compact set of a metric space.
7. If $\gamma(t) = e^{it}$ where $0 \leq t \leq 2\pi$. Show that γ is rectifiable.
8. Define uniform continuity of a function.
9. Define convex set. Give an example.
10. Is the subset of a connected set always connected? Justify.
11. What you mean by discontinuity of second kind. Give an example.
12. Is set of rational numbers connected? Justify your answer.
13. Define refinement of a partition. Show that $\underline{\int_a^b} f\, d\alpha \leq \overline{\int_a^b} f\, d\alpha$
14. With an example define uniform convergence of sequence of functions.

(14 x 1 = 14 Weightage)

PART B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Discuss the continuity of the function $f(x) = \begin{cases} 1, & x\ is\ rational \\ 0, & x\ is\ irrational \end{cases}$
16. Show that if f is continuous on $[a, b]$ then $f \in R(\alpha) on\ [a, b]$
17. State and prove fundamental theorem of calculus.
18. Show that C(X) is a complete metric space.

19. Prove that Let A be the set of all sequences whose elements are 0 *and* 1, then A is uncountable.
20. State and prove Taylor's theorem.
21. Show that compact subsets of a metric space is closed
22. Show that if f is continuous mapping of a metric space X into a metric space Y and if E is connected subset of X, then *f(E)* is connected.
23. State and prove Cauchy criterion for uniform convergence.
24. Prove that a set E is open if and only if its compliment is closed.

(7 x 2 = 14 Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. State and prove L'Hospital's rule.
26. Prove that every bounded infinite subset of R^k has a limit point in R^k
27. If γ' is continuous on $[a, b]$ then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt$
28. If f is a continuous one-one mapping of a compact metric space X onto a metric space Y, then prove that the inverse mapping f^{-1} defined by $f^{-1}(f(x)) = x$ is a continuous mapping of Y onto X.

(2 x 4 = 8 Weightage)

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