18P102 (Pages: 2) Name: ……………………

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C02 / CC17P MT1 C02 – LINEAR ALGEBRA

(Mathematics)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Verify whether $\{(1,2,3),(1,3,1)\}$ is a basis for R^3.
2. Find the dimension of the space of all nxn diagonal matrices over R.
3. Let V = R^2 and $W_1 = \{(x,0) : x \in R\}$. Find a subspace W_2 of V such that $V = W_1 \oplus W_2$
4. Find the characteristic polynomial of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
5. Find the null space of the linear transformation $\mathrm{T}: \mathrm{R}^2 \to \mathrm{R}^2$ where $\mathrm{T(x,\ y)} = \mathrm{(y,\ x)}$
6. Let T be a linear operator on a vector space V. Verify whether Im (T) is invariant subspace for T.
7. Define non-singular transformation. Show that a linear transformation T is non singular iff T is one-one.
8. Find the co-ordinate vector of $(1,2,3) \in R^3$ with respect to the ordered basis $\{(1,2,0),(1,1,0),(0,1,1)\}$.
9. Let $\mathrm{W} =$ span $\{(1,1,0),(1,0,1)\}$. Let F be defined by $\mathrm{F(x,y,z) = x - y - z}$. Verify whether F belongs to W^o.
10. Give an example of a linear functional on R^3.
11. Show that $\mathrm{T(x,y) = (2x + y, y)}$ is diagonalizable.
12. Verify whether $\mathrm{T}: \mathrm{R}^2 \to \mathrm{R}^2$ defined by $\mathrm{T(x,y) = (2x + y, 0)}$ is a projection.
13. Define an inner product space and write an example for the same.
14. Find the orthogonal complement of $W = \{(x,x): x \in R\}$ in R^2.

(14 x 1 = 14 Weightage)

PART B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Prove that the intersection of two subspaces of a Vector Space is a subspace of the Vector Space. Is the union of any two subspaces again a subspace? Justify your claim.

16. Define characteristic polynomial of a matrix. Prove that similar matrices have the same characteristic polynomial
17. Show that a finite dimensional vector space V is linearly isomorphic to its second dual.
18. Show that the row space of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ is R^3
19. Does there exist a linear transformation from R^3 in to R^2 such that $T(1,-1,1) = (1,0)$ and $T(1,1,1) = (0,1)$?
20. Let F be a field and let T be the operator on F^2 defined by $T(x_1, x_2) = (x_1, 0)$. Find the matrix of T relative to the standard basis of F^2.
21. Prove that if W_1 and W_2 are subspaces of a finite dimensional vector space then $W_1 = W_2$ if and only if $W_1^o = W_2^o$.
22. Let B= $\{\alpha_1, \alpha_2, \alpha_3\}$be the basis for C^3, defined by $\alpha_1 = (1,0,-1), \alpha_2 = (1,1,1)$, $\alpha_3 = (2,2,0)$. Find the dual basis of B.
23. Let V and W be two finite dimensional vector spaces over the field F and let T be a linear transformation from V into W. Then prove that Rank(T^t) = Rank(T)
24. Prove that every projection is diagonalizable.

(7 x 2 = 14 Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Define dimension of a vector space. Show that if W_1, W_2 are subspaces of a finite dimensional vector space, then

$$dim(W_1 + W_2) = dim\, W_1 + dim\, W_2 - dim(W_1 \cap W_2)$$

26. Let V be an finite dimensional vector space over the field F and let T be a linear operator on V. Then T is diagonalizable if and only if the minimal polynomial for T has the form $P(x) = (x - c_1)(x - c_2) \ldots\ldots (x - c_k)$ where $c_1, c_2, \ldots\ldots c_k$ are distinct elements of F.
27. Let V be a vector space of dimension n and W a vector space of dimension m. Show that L(V,W) is a vector space of dimension mn.
28. (a) Prove that every finite dimensional inner product space has an orthonormal basis.
    (b) Apply Gram-Schmidt orthogonalization process to $(1,0,1), (1,0,-1)$, $(0,3,4)$ to obtain an orthonormal basis for $\mathbf{R}^3$ with the standard inner product.

(2 x 4 = 8 Weightage)

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