18P166 (Pages: 2) Name: ……………………

Reg. No…………..……..

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P ST1 C05 – DISTRIBUTION THEORY

(Statistics)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Write the probability density function of bivariate normal distribution.
2. Let $X_1 \sim NB(\alpha_1, p)$, $X_2 \sim NB(\alpha_2, p)$. Find the distribution of $Y = X_1 + X_2$
3. Define truncated distribution. Write the probability mass function of truncated Poisson distribution.
4. If $X \sim U(0,1)$, Write the probability density function of $Y = -\log X$
5. Define order statistic. Write the general form of the probability density function of r^{th} order statistic $X_{(r)}$
6. Define mixture distributions. Discuss the practical situations where mixture distributions are appropriate.
7. Discuss the properties of location-scale family of distributions.
8. Discuss the interrelationship between t, F, Chi square statistic.
9. If 'F' has F distribution with degrees of freedom $n_1 = n_2 = n$, show that its median M is one.
10. Identify two distributions each of which

    a) belongs to power series family b) does not belong to the power series family
11. Let X be a random variable with a continuous distribution function F. Then show that $F(x)$ has the uniform distribution on [0, 1]
12. Let X and Y be independent random variables. Show that $P_{X+Y}(t) = P_X(t).P_Y(t)$, where $P(.)$ is the PGF of the random variable.

(12 x 1 = 12 Weightage)

Part B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. If X is a random variable that takes only positive values, then for any value $a > 0$, Show that $P\{X \geq a\} \leq \frac{E(X)}{a}$
14. If X and Y are independent Poisson random variables with respective means λ_1 and λ_2 Find the distribution of $X\ /(X+Y) = n$

15. Suppose the joint density of X and Y is given by

$f(x,\ y) = \begin{cases} 6xy(2-x-y), & 0 < x < 1, 0 < y < 1 \\ 0\ elsewhere \end{cases}$ Find $E(X|Y=y)$

16. Show that $X_{1,}X_{2,} \dots, X_{n,}$ are identically and independently distributed $Beta\ (\alpha, 1)$ random variables if and only if $M_{n,} \sim Beta\ (\alpha n, 1)$, where $M_n = \text{Max}(X_{1,}X_{2,} \dots, X_{n,})$

17. Define non central 't' distribution. Show that square of non central 't' follows non central 'F' distribution.

18. Define Weibull distribution. Obtain the distribution of U where U = $\text{Min}(X_{1,}X_{2,} \dots, X_{n,})$ if $X_{i,}'s$ independently distributed according to standard Weibull .

19. Define hypergeometric distribution. Find its mean and variance.

20. For any integer valued random variable, show that $\sum_{n=0}^{\infty} s^n P(X \leq n) = \frac{P(s)}{1-s}$, where $P(S)$ is the PGF of X

21. Let X and Y have independent gamma distribution with parameters μ and ν repectively. Find the joint distribution U + X + Y and $W = \frac{X}{Y}$

22. If X and Y are independent exponential random variables with parameter one, show that $\frac{X}{X+Y}$ has $U(0,1)$ distribution.

23. If X and Y are independent random variables with density function $f(x) = e^{-x}$, $0 < x < \infty$. Show that $Z = \frac{X}{Y}$ has F- distribution.

24. Derive Pearson Type III distribution. Also obtain it as a generalization of gamma distribution.

(8 x 2 = 16 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. In sampling from normal population. Prove that the sample mean $\bar{X}$ and sample variance S^2 are independently distributed.

26. Describe log normal distribution. Obtain its moment generating function and determine its coefficient of variation.

27. Define Non central Chi-square distribution and derive the PDF. Also find its mean and variance.

28. i) Show that $Var(X) = E\big(Var(X|Y)\big) + Var\big(E(X|Y)\big)$

    ii) Derive the joint distribution of $X_{(r)}$ and $X_{(s)}$, the r^{th} and s^{th} order statistics.

(2 x 4 = 8 Weightage)

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