18P162 (Pages: 2) Name: …………………….

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P ST1 C01 – MEASURE THEORY AND INTEGRATION

(Statistics)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. When do you say a function is Riemann-Stieltjes integrable? Give Example.
2. What are the sufficient conditions for Riemann-Stieltjes integrability?
3. Distinguish between charge and measure.
4. If $E \in X$, Show that characteristic function is measurable.
5. State Convergence in almost every where.
6. Define Lebesgue integral of non-negative function.
7. State Hahn decomposition theorem.
8. What is total variation?
9. Define continuity of measures.
10. Give any three properties of Outer measure.
11. What is the sufficient condition for a set A subset of X is μ* measurable
12. Define product measure. Is x-section is measurable?

(12 x 1 = 12 Weightage)

Part B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. Show that $U(\bar{P}, f, \alpha) \leq U(P, f, \alpha)$, if $\bar{P}$ is a refinement of P.
14. Suppose that $\lim_{n\to\infty} f_n(x) = f(x), x \in E$ and $f_n \to f$ uniformly on E. Then $\lim_n \lim_t f_n(t) = \lim_t \lim_n f_n(t)$
15. If φ and η are non-negative functions then show that $\int(a\phi + b\eta)\, d\mu = a\int \phi d\mu + b\int \eta d\mu$.
16. If a sequence $\{f_n\}$ be a sequence of measurable functions. Then show that $\lim_n Inf f_n(x) = f(x), x \in E$ is measurable.
17. State and prove Lebesgue Dominated convergence theorem.

18. State and prove Holder's inequality.

19. State and prove Jordan Decomposition theorem.

20. Show that if a sequence $\{f_n\}$ convergence to f in L_p, then it is Cauchy in measure.

21. State the sufficient condition for countable additively of μ* measure. Prove it.

22. State and prove Fubini's theorem.

23. State and prove Monotone Class Lemma.

24. Define X-section. Show that f_x is measurable for a bivariate measurable function f(x,y).

(8 x 2 = 16 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. State and Prove Lebesgue Decomposition theorem.

26. Derive the relationship between integration and uniform continuity.

27. State and Prove Radon Nikodym Theorem.

28. State and prove Caretheody Extension theorem.

(2 x 4 = 8 Weightage)

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