**19P157** (Pages: 2) Name: ……….……………

Reg. No…….……………...

**FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019**

(CUCSS PG)

**CC19P MST1 C02 – ANALYTICAL TOOLS FOR STATISTICS II**

(Statistics)

(2019 Admission Regular)

Time: Three Hours Maximum : 30 Weightage

**PART-A**

Answer any ***four*** questions. Each question carries 2 weightage.

1. (i) Define vector space and subspace of a vector space.

   (ii) Show that intersection of two sub spaces of a vector space is also a vector space.
2. Define direct sum and complement of subspaces. Write the necessary and sufficient conditions for sum of two subspaces to be direct sum.
3. Check whether the following mapping is linear or not.

   $F : R^3 \rightarrow R^2$ is defined by $F(x, y, z) = (x + y + z, 2x - 3y + 4z)$
4. Define idempotent matrix and nil potent matrix. Show that for an idempotent matrix,

   $r(A) + r(I - A) = n$
5. Verify Cayley – Hamilton theorem for $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & -3 & 0 \\ 1 & 1 & -1 \end{bmatrix}$ and hence find $A^{-1}$
6. Show that the characteristic vectors corresponding to distinct characteristic roots are independent.
7. Define quadratic form. Write the classification of quadratic form.

**(4 x 2 = 8 Weightage)**

**PART-B**

Answer any ***four*** questions. Each question carries 3 weightage.

8. Find a basis and dimension of the solution space W of the homogeneous system

   $x + 2y - 2z + 2s - t = 0, \quad x + 2y - z + 3s - 2t = 0, \quad 2x + 4y - 7z + s + t = 0$
9. Let V be a finite dimensional vector space and let $(v_1, v_2, \ldots, v_n)$ be any basis. Show that (i) If a set has more than n vectors; then it is linearly dependent.

   (ii) If a set has fewer than n vectors; it does not span V.
10. Reduce the matrix A to normal form and find the rank $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

11. (i) Show that $r(AB) \le \min(r(A), r(B))$

(ii) Prove that the nullity of the product of two square matrices is at least as great as the nullity of either factor.

12. Show that algebraic multiplicity of an eigen value of a matrix is always greater than or equal to geometric multiplicity.

13. Find Jordan canonical form for A = $\begin{bmatrix} 2 & 2 & 3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$

14. Show that Moore - Penrose inverse is exist and is uniquely defined.

**(4 x 3 = 12 Weightage)**

## PART-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. (a) If V is a vector space over a field F and W is a subspace V, prove that

$$\dim(V/W) = \dim(V) - \dim(W)$$

(b) Compute an orthonormal basis of $P_3(t)$ where one of its basis is given by $\{1, t, t^2, t^3\}$ and $\langle f, g\rangle = \int_{-1}^{1} f(t)g(t)\,dt$

16. Let A= $\begin{bmatrix} 4 & 1 & 6 & 0 \\ 0 & 1 & 2 & -4 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ Show that C (A) = $\{(\alpha, \beta, \gamma): \alpha - \beta - 4\gamma = 0\}$,

$R(A) = \{(\alpha, \beta, \gamma, \delta): 3\alpha - 2\gamma - \delta = 0\}$. Also obtain the basis and dimension of C(A) and R(A). C(A) and R(A) denotes column space of A and row space of A respectively.

17. If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are eigen values of a square matrix A not necessarily distinct then Prove that

(i) Product of eigen values = determinant of A

(ii) Sum of eigen values = Trace of A

18. Reduce the quadratic form $3x_1^2 + 2x_1x_2 + 2x_1x_3 + 4x_2x_3$ into a canonical form by the method of orthogonal reduction. Also find the rank and signature.

**(2 x 5 = 10 Weightage)**

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