19P104A (Pages: 2) Name: ……….……………

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C05/CC17P MT1 C05 – DISCRETE MATHEMATICS

(Mathematics)

(2015 to 2018 Admissions)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define totally ordered set with example.
2. Is there a Boolean algebra with 9 elements? Justify your answer.
3. Prepare the table of values of the following function $f(x_1, x_2, x_3) = x_1{}'x_2(x_1{}' + x_2 + x_1)$
4. Let $(X, +, ., ')$ be a finite Boolean algebra, then prove that every two distinct atoms of X are mutually disjoint.
5. Prove that in any graph G, the number of vertices of odd degree is even.
6. Define dual of a graph. Draw the dual graph of C_5
7. Prove that a connected graph G is a tree if and only if every edge of G is a cut edge of G
8. Find $k(G), \lambda(G)$ & $\delta(G)$ where $G = K_5$
9. Prove that a graph is planar if and only if it is embeddable on a sphere.
10. Prove that the Petersen graph is nonplanar.
11. Prove or disprove: Let G be a simple connected graph with $n(G) \geq 3$, then G has a cut edge if and only if G has a cut vertex.
12. Differentiate between Dfa and Nfa.
13. Show that the language L $=\{awa/w \in \{a, b\}^*\}$ is regular.
14. Let G($\{S\}, \{a, b, \}, S, P$) be a grammar with productions P given by S → aA , A → bS, $S \to \lambda$. Give a simple description of the language generated by G

(14 x1=14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Find the DNF and CNF of the Boolean function $f(a, b, c) = a + b + c'$
16. Let (X$, +, ., '$) is a finite Boolean algebra then prove that the relation ≤ defined by $x \leq y$ if $x.y' = 0$ makes the underlying set of Boolean algebra into a lattice. Moreover 0 and 1 are the minimum and maximum elements of this lattice.

17. Prove that the number of edges in a tree with n vertices is $n-1$. Conversely, a connected graph with n vertices and $n-1$ edges is a tree.
18. State and prove Euler's formula.
19. Prove that $K_{3,3}$ is nonplanar.
20. Prove that in a graph an edge is a cut edge if and only if it belongs to no cycle.
21. Prove that every connected graph contains a spanning tree.
22. Construct the DFA of all strings ending with aab, where $\sum = \{a, b\}$
23. Prove that $(UV)^R = U^R V^R$ for all strings U, V and of any length.
24. Find the grammar that generates L = $\{a^n b^{n+1}\ ; n \geq 0\}$

(7 x 2=14 Weightage)

Part C

Answer any ***two*** Questions. Each question carries 4 weightage.

25. Prove that a graph is bipartite if, and only if, it contains no odd cycles.
26. Let L be the language accepted by a NFA $M_N = (Q_N, \sum \delta_{N\,,}\, q_0\,, F_N)$. Prove that there exist a DFA $M_D = (Q_D, \sum \delta_{D,}\ q_0,\ F_D)$ such that $L = L(M_D)$
27. For a connected graph G, prove that the following statements are equivalent:
    (a) G is Eulerian.
    (b) The degree of each vertex of G is an even positive integer.
    (c) G is an edge-disjoint union of cycles.
28. a) Let (X, ≤) be a poset and A be a non-empty finite subset of X. Prove that A has at least one maximal element.
    b) Let $(\,X, +, .\,, ')$ be a Boolean algebra. Prove that $x + x.y = x$ for $all\ x, y \in\ X$

(2 x 4 = 8 Weightage)

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