19P103A (Pages: 2) Name: ………..……………

Reg. No……….…………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C03 / CC17P MT1 C03 – REAL ANALYSIS-I

(Mathematics)

(2015 to 2018 Admissions)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question has 1 weightage.

1. Give an example of an open cover of the segment $(0,1)$ which has no finite sub cover.
2. Is every point of every open set $E \subset R^2$ a limit point of E ? Justify.
3. Prove that E is open if and only if $E^o = E$
4. Prove or disprove: Every continuous function is an open mapping.
5. Let f be a continuous real function on a metric space X. Let $Z(f)$ be the set of all $p \in X$ at which $f(p) = 0$. Prove that $Z(f)$ is closed.
6. Discuss the continuity of the function $f(x) = \begin{cases} x, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$
7. Discuss the differentiability of $|x|^3$ in R
8. State intermediate value theorem. Is the converse true?
9. Define refinement of a partition. Show that $\underline{\int_a^b} f\, d\alpha \leq \overline{\int_a^b} f\, d\alpha$
10. Show that if $f_1(x) \leq f_2(x) on\ [a,b]\, , then \int_a^b f_1\, d\alpha \leq \int_a^b f_2\, d\alpha$
11. If $\gamma(t) = e^{2it}$ where $0 \leq t \leq 2\pi$. Show that γ is rectifiable.
12. Suppose $\{f_n\}$ is an equicontinuous sequence of function on a compact set K and $\{f_n\}$ converges pointwise on K. Prove that $\{f_n\}$ converges uniformly on K
13. With an example define uniform convergence of sequence of functions.
14. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

(14 × 1 = 14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Construct a compact set of real numbers whose limit points form a countable set.
16. Prove or disprove: "$A\ set\ of\ finite\ points\ has\ no\ limit\ point$".

17. Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y
18. Show that if *f* is continuous mapping of a metric space X into a metric space Y and if E is connected subset of X, then *f(E)* is connected.
19. If f is differentiable on $[a,b]$. Then show that f' cannot have any simple discontinuities on $[a,b]$
20. Define local maximum of a function. Show that first derivative of this function is zero at local maximum point.
21. If γ' is continuous on $[a,b]$ then prove that γ is rectifiable.
22. Show that if f is continuous on $[a,b]$ then $f \in R(\alpha)$ on $[a,b]$
23. State and prove Cauchy criterion for uniform convergence.
24. Show that set of all complex valued continuous bounded functions with domain X is a complete metric space.

(7 × 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Is the set of real numbers is countable? Justify.
26. Show that monotonic functions have no discontinuities of the second kind.
27. State and prove L'Hospital's rule.
28. Show that there exists a real continuous function on the real line which is nowhere differentiable.

(2 × 4 = 8 Weightage)

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