19P105   (Pages: 2)   Name: ………..……………

Reg. No……..….………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCSS PG)

CC19 MTH1 C05 – NUMBER THEORY

(Mathematics)

(2019 Admission Regular)

Time: Three Hours   Maximum: 30 Weightage

Part A

Answer *all* questions. Each question carries 1 weightage.

1. Define Euler Toitent function $\varphi(n)$ and show that if $a|b$ then $\varphi(a)|\varphi(b)$

2. Prove that $[x + n] = [x] + n$, where $x \in R, n \in N$

3. Derive Selberg identity.

4. Define Chebyshev's functions $\psi(x)$ and $\vartheta(x)$ and show that $\psi(x) = \sum_{m \le \log_2 x} \vartheta(x^{1/m})$

5. Calculate the highest power of 10 that divides 1000!

6. Prove that the Jacobi symbol is a completely multiplicative function.

7. In the 27-letter alphabet (blank = 26) use the affine enciphering transformation with key a = 13, b = 9 to encipher the message "HELP ME".

8. What is a trapdoor function?

(8 × 1= 8 Weightage)

Part B

Answer any *two* questions from each unit. Each question carries 2 weightage.

UNIT I

9. Show that if $n \ge 1$ then $\sum_{d/n} \varphi(d) = n$

10. Define Mangoldt function. Show that $n \ge 1$ then $\log n = \sum_{d/n} \Lambda(d)$

11. Prove that $\forall\ x\ \ge 1, \left| \sum_{n \le x} \frac{\mu(n)}{n} \right| \le 1$

UNIT II

12. State and prove Abel's identity.

13. For $x \ge 2$, prove that $\vartheta(x) = \pi(x) \log x - \int_2^x \frac{\pi(t)}{t} dt$ and $\pi(x) = \frac{\theta(x)}{\log x} + \int_2^x \frac{\theta(t)}{t \log^2 t} dt$

14. Prove that $\lim_{x \to \infty} \left(\frac{M(x)}{x} - \frac{H(x)}{x \log x} \right) = 0$

UNIT III

15. If p is an odd prime, prove that $(2|p) = (-1)^{\frac{(p^2-1)}{8}}$

16. Determine those odd primes for which 3 is a quadratic residue and those for which it is a non-residue.

17. Solve the following system:

$$x + 3y \equiv 1 (mod\ 26)$$
$$7x + 9y \equiv 1 (mod\ 26)$$

(6 × 2 = 12 Weightage)

Part C
Answer any *two* questions. Each question carries 5 weightage.

18. (a) State and prove Euler's summation formula.

   (b) If $x \geq 1$, prove that $\sum_{n \leq x} \frac{1}{n} = \log x + C + O\left(\frac{1}{x}\right)$ where C is the Euler's constant.

19. (a) State three equivalent forms of prime number theorem.

   (b) Prove that that $\lim_{x \to \infty} \frac{\pi(x)\ log\ x}{x} = 1$ if and only if $\lim_{x \to \infty} \frac{\pi(x)\ log\ \pi(x)}{x} = 1$

20. State and prove Gauss' lemma.

21. (a) What is the difference between public key and private key cryptosystem.

   (b) Working in 26-letter alphabet use the matrix $A = \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix}$ to encipher the message "NOANSWER".

(2 × 5 = 10 Weightage)

****